Collusion, Incentives and Information: The Role of Experts in Corporate Governance*

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Abstract

The paper shows that the severer moral hazard problems of the CEO and poorer ability of the outside expert not only increase information rent to the CEO on their own but also worsen the problem of collusion between them. The owners then incur extra costs, attributed to increased incentive and severance payments to the CEO. The expert’s incentives to acquire reputation or the presence of public signals mitigate such problems but cannot undo them completely. The paper derives empirical predictions on experts’ turnovers, fees, their impacts on corporate restructuring and draws implications on the thinness of market for experts.

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1 Introduction

The occurrence of the series of scandals in these years, involving firms like Enron, Tyco, Global Crossing, put much of the media spotlight on the expert firms, like investment banks, audit firms, consultancies, which typically gather, process and interpret valuable information for their client firms.\(^1\) Also a large number of academic studies documented overt or covert collusion between the expert firms and the insiders of their client firm such as the CEO/CFO/board members.\(^2\) Many safeguards that are popularly expected to protect shareholders from fraud and malfeasance, such as the reputation of the expert firms and the much availability of public information, failed the expectation. In light of these contemporary problems related to collusion and fraud, this paper aims to undertake two investigations. Firstly, it investigates the link between the composition of the CEO’s compensation package, his incentives to collude with the expert for suppressing bad news, and the costs such collusion imposes on the shareholders of the firm. Secondly, the paper investigates the impacts of the two aforementioned means of checks and balances: (a) the concern of the expert about her reputation of being non-collusive and honest; and (b) the presence of public information.

We address these issues in a standard framework where the shareholders hire a CEO for running a project, and an expert for advisory services, who, depending on the exact situation, could be an audit firm, an investment banker, a consultant, or an expert board member. Before the project is completed, the expert digs out a signal indicative of its profitability. When the signal is low, then it is better to liquidate the project than to continue it, while the reverse is true when the signal is high. The shareholders

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\(^1\)A recent example is concerned with the auditors of Lehman Brothers. Even though the audit firm for the company was spared further investigations, it was reported, "(T)he judge said this week that it appeared Lehman had violated Generally Accepted Accounting Principles, or GAAP, even if it was in technical compliance with accounting rules." (see "Lehman Case Hints at Need to Strengthen Audit Rules," New York Times, 28, July, 2011)

do not observe either the CEO’s effort or the realization of this signal. The information conveyed by the signal can be used not only to improve decision on the interim stage but also to incentivize the CEO. If truthfully reported by the expert, this signal would always create value for the owners of the firm. But the CEO might have incentives to collude with the expert to distort the report, which will reduce the value of the expert’s service to the shareholders.

We build on the model of expertise developed in Martmort and Gromb (2007) and embed in the standard framework of managerial moral hazard as in Almazan, Banerji and Demotta (2008), Laffont and Martimont (2003), Tirole (2001, 2006), and derive the following main results.

First, we show that even if the CEO and expert can perfectly collude, the collusion does not always destruct value. Under certain circumstances, it can be checked if the firm designs proper contracts to the CEO and the expert, with the help of the two means of checks and balances. Two uniform patterns appear in all the three cases we consider (i.e. the static case, the dynamic case where the expert interacts with the firm over time, and the case with the public information): the severer the moral hazard problems of the CEO, or the less informative the expert, the more likely the problem of collusion bites. Therefore, not only the CEO with severer moral hazard problems and the expert of lesser ability increase the “information rent” to the CEO and thus destroy the value of the firm separately on their own, but also they together are more likely to engage into value-destructing collusion.

Second, we show that the additional costs incurred by the collusion take two forms: (a) incentive based payments (stock or option holdings) and (b) severance payments. Intuitively, to elicit truthful revelation of bad news, the firm has to reward the collusive alliance upon liquidation, in terms of the severance payments, which, however, dilute the CEO’s incentives to exert greater effort. To elicit effort, consequently, the firm has to reward him with even larger incentive payments.

Third, we show that the concern of the expert about her reputation of being honest, when she is in repeated interactions with the firm, helps curb the collusion problem, but fails to undo it, for two reasons. One, the expert is able to establish reputation only if her ability of processing information is beyond a threshold. The other, even
when she is able to, the firm wants to resort to reputational mechanisms for deterring collusion only if her ability is beyond another, still higher, threshold. These two results imply that market for reputable experts is likely to be thin.\footnote{This is corroborated with findings by Rau (2000) and Ross (2010) on relatively smaller number of reputable expert firms.}

Fourth, public information, such as media reports or analyst opinions, affects the value of the firm via two channels: it is informative about the CEO’s behavior and can thus be used to reduce the cost of incentivizing him; and it curbs the collusion problem when the signal is weakly informative. A very strong public signal, however, could surprisingly exacerbate the problem. We also show that collusion stresses the firm to the most when the public information pictures a rosy prospect. This is consistent with the empirical finding by Wang, Winton and Wu (2010).

Moreover, regarding expert firms, we predict that while the market for reputable experts is thin, they have longer relationship with their clients and add a higher value to them than non-reputable experts. The non-reputable experts, by contrast, are substitutable at any period and thus their business relationship may feature a high turnover rate, for which Srinivasan (2005) and Yermack (2004) provide supportive evidence.

The paper contributes to the literature that examines collusion in a wide range of contexts; see Barron and Sappington (1997), Baliga and Sjostrom (1998), Gromb and Martimont (2007), Khalil et al. (2010), Kofman and Lawerree (1993), Laffont (2001), and Tirole (1986), among others. Most of these papers study collusion in relation to bribery and extortions, efficiency of internal organization of the firm, tax evasion, auditing, or the design of institutions in a political economy context. Distinctively, our paper underlines the interaction between collusion, incentives, and provision of information. Furthermore, the present paper examines the two means of checks and balances (i.e. the reputational concern of the expert and the presence of public information), whereas those papers do not.

Also, the paper is related to the literature that discusses severance payments in the context of both symmetric and asymmetric information; see, Almazan and Suarez (2003), Eisfeldt and Rampini (2008), Inderst and Mueller (2010), and Levitt and Snyder (1997). Like most of these studies, our paper also shows that severance payments are
used as a tool for extracting bad news. But different from them, in which the CEO is in charge of both the task of exerting effort and that of collecting and reporting the interim signal, our paper splits the two tasks allocating the former to the CEO and the latter to the expert, with each agent under a different incentive scheme. This enables us to provide new insights regarding the biting of collusion and to look into the two means of checks and balances.

The organization of the paper is as follows: In Section 2, we introduce the basic model where we examine the contracting between the shareholders, the CEO, and the expert in a static setting. In Section 3 we extend the basic model into a setting of repeated interactions to examine the reputational mechanism as a means of checks and balances. In Section 4, we add public information to the basic model and examine its efficacy in curbing collusion. In Section 5 we make concluding comments. All proofs are relegated to Appendix.

2 The Basic Model

In the model the shareholders of a firm, referred to as "the firm", hire a CEO to run a project and an expert for advisory services. All the agents are risk neutral. Both the CEO and the expert enjoy limited liability in the sense that the transfers specified in their respective contracts can not be negative. The project could be liquidated before its completion, if in the midway the expert receives a bad signal about its prospect. The liquidation generates a deterministic cash flow \( L \). If it is not liquidated, the project has an uncertain return \( R \in \{y, 0\} \).

There are thus three relevant dates, with \( t = 0 \) for contracting, \( t = 1 \) for the expert’s evaluation and the interim decision (to liquidate or to continue the project), and \( t = 2 \) for the realization of the outcome of the project if it is continued at \( t = 1 \). Events are illustrated in Figure 1.

At \( t = 0 \), the firm offers contingent contracts (elaborated below) to the CEO and the expert. The CEO then chooses either to shirk or to exert effort at cost \( B \). This choice is his private information. If he exerts effort, the project will reach a good (or viable) state at \( t = 1 \) with probability \( q \), and a bad (or inviable) state with probability
If he shirks, the probability of the project reaching the good state lowers to $q - \Delta$, with $\Delta > 0$. If the project is in the good state and is not liquidated at $t = 1$, it will succeed (namely yielding $y$) with probability $s_g < 1$. The probability of success in the bad state is $s_b$. We assume $s_by < L < s_gy$ and that the firm prefers to incentivize the CEO to exert effort.

At $t = 1$, the expert evaluates the state of the project and reports her evaluation to the firm. At a cost of $B_x$, which could represent the opportunity cost, she receives a private and noisy signal which is either high or low, representing good news or bad news, respectively. If the signal is low, then the continuation value of the project is less than its liquidation value and the project shall be liquidated. The reverse is true when the expert receives a high signal. We assume that the expert’s signal, though soft in nature (namely, manipulatable to the shareholders), is perceived by the CEO from his experience of running the project. This assumption enables us to avoid the problem of collusion under asymmetric information between the CEO and the expert.

Formally, let $\tilde{m} = h, l$ denote respectively the high or low signal. Conditional on the state of the project, $\tilde{m}$ is distributed as follows:

$$
\Pr(\tilde{m} = h|\text{Good}) = \Pr(\tilde{m} = l|\text{Bad}) = \lambda > 0.50
$$

So $\lambda$ measures the informativeness of the expert’s signal or the capability of her.

Upon the report by the expert, the firm decides whether to liquidate or continue the project, with liquidation yielding value $L$. And we assume:

$$
\begin{align*}
    p_hy > L > p_ly
\end{align*}
$$

where $p_h$ ($p_l$) is the posterior probability of success conditional on $\tilde{m} = h$ ($\tilde{m} = l$). That is, $p_h = e_h/q_h$ and $p_l = e_l/q_l$, where

$$
e_h \equiv q\lambda s_g + (1 - q)(1 - \lambda)s_b
$$

is the ex ante probability of event that $\tilde{m} = h$ and the project succeeds (if not liquidated middle way), and

$$
q_h \equiv q\lambda + (1 - q)(1 - \lambda)
$$

is the ex ante probability of event that $\tilde{m} = h$. Similarly we define $e_l$ and $q_l$. Certainly, $q_h + q_l = 1$. 


Assumption (1) requires the expert’s signal is informative enough:

\[ \lambda > \max \left( \frac{1}{2}, \frac{(1-q) \left( \frac{L}{y} - s_b \right)}{q(s_g - \frac{L}{y}) + (1-q) \left( \frac{L}{y} - s_b \right)}, \frac{q(s_g - \frac{L}{y})}{q(s_g - \frac{L}{y}) + (1-q) \left( \frac{L}{y} - s_b \right)} \right) \]  

(2)

At \( t = 2 \), the cash flow of the continued project, either \( y \) or \( 0 \), is realized and publicly observed. The expert and the CEO are paid according to the contracts drawn up at \( t = 0 \) and the residual goes to the shareholders.

A contract to the CEO is represented by \( \{w_l, w_h, w_f\} \), where \( w_l \) is the payment to him when the expert reports \( \tilde{m} = l \) and thus the project is liquidated and \( w_h(w_f) \) is the payment to him when she reports \( \tilde{m} = h \) and the continued project succeeds (fails). Similarly, a contract to the expert is represented by \( \{x_h, x_f, x_l\} \).

**A. The Social Value of the Expert**

The expert’s information improves the interim decision. We assume that without her service, the project is always continued at \( t = 1 \), that is, \( (q s_g + (1-q)s_b)y > L \). Then, the improvement in the interim decision by the expert comes from liquidation she recommends and the value of her service, denoted by \( \Omega \), equals \( q(1-\lambda)(L - s_g y) + (1-q)\lambda(L - s_b y) \): either she receive a low signal in the good state and thus might make an error in advising liquidation, causing a loss of value equal to \( L - s_g y \), which is represented by the first term; or she receives a low signal in the bad state where liquidation saves value \( L - s_b y \), which is captured by the second term.

The cost of her service is \( B_x \). The paper assumes her service is socially efficient:

\[ \Omega - B_x > 0. \]

**B. The Contracting Problem of the Firm**

The contracts to the CEO and the expert designed by the firm at \( t = 0 \) must induce (a) the CEO to exert effort at \( t = 0 \), (b) the expert to disclose truthfully her information at \( t = 1 \), and (c) the expert and the CEO not to collude to distort the disclosure at

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\(^4\)By revelation principle, without loss of generality we confine ourselves to the case where the expert’s signal is truthfully reported in equilibrium; thus the firm will liquidate the project at \( t = 1 \) if \( \tilde{m} = l \) is reported, and continue it if \( \tilde{m} = h \) is reported – then it either succeeds or fails.
$t = 1$. Optimal contracts minimize the expected costs of compensating the CEO and the expert subject to these incentive compatibility, truth-telling, collusion proofness and participation constraints, detailed below. Recall that $e_h$ denotes the ex ante probability of event that $\tilde{m} = h$ and the project succeeds, $q_h$ the ex ante probability of event $\tilde{m} = h$, and thus $q_h - e_h$ is the ex ante probability that $\tilde{m} = h$ and the project fails; and $q_l = 1 - q_h$ is the probability that $\tilde{m} = l$. The firm’s problem is:

$$\min_{\{w_h, w_f, w_l, x_h, x_f, x_l\}} e_h(w_h + x_h) + (q_h - e_h)(w_f + x_f) + q_l(w_l + x_l),$$

subject to

(a) CEO moral hazard constraint:

$$e_h w_h + (q_h - e_h) w_f + q_l w_l \geq B + [(q - \Delta) \lambda s_g + (1 - (q - \Delta))(1 - \lambda) s_b] w_h + ((q - \Delta) \lambda (1 - s_g) + \{1 - (q - \Delta)\}(1 - \lambda)(1 - s_b)] w_f + [(q - \Delta)(1 - \lambda) + (1 - (q - \Delta))] w_l$$

(b) Adverse selection constraints of the expert:

$$p_h x_h + (1 - p_h) x_f \geq x_l \quad x_l \geq p_l x_h + (1 - p_l) x_f$$

(c) Collusion proofness constraints:

$$p_h(x_h + w_h) + (1 - p_h)(x_f + w_f) \geq x_l + w_l \quad x_l + w_l \geq p_l(x_h + w_h) + (1 - p_l)(x_f + w_f)$$

(d) Limited liability constraints: $w_h \geq 0, w_f \geq 0, w_l \geq 0, x_h \geq 0, x_f \geq 0$ and $x_l \geq 0$.

(e) The individual rationality (IR) constraint of the expert:

$$e_h x_h + (q_h - e_h) x_f + q_l x_l \geq B$$

(f) The IR constraint of the CEO:

$$e_h w_h + (q_h - e_h) w_f + q_l w_l \geq B.$$
Condition (4) ensures that at \( t = 1 \) the expert alone does not gain by mis-reporting \( m = h \) as \( m = l \). Telling the truth leads the project continued, whereby she expects to obtain \( p_h x_h + (1 - p_h) x_f \), whereas she would obtain \( x_l \) if lying that \( m = l \). Similarly, (5) ensures no gain for her to mis-report \( m = l \).

Condition (6) ensures that at \( t = 1 \) the CEO and the expert jointly do not gain by mis-reporting \( m = h \) as \( m = l \). If \( m = h \) is truthfully reported, thus the project continued, together they expect to get \( p_h (x_h + w_h) + (1 - p_h) (x_f + w_f) \), whereas they get \( x_l + w_l \) by fraudulently reporting \( m = l \). Similarly, Condition (7) ensures no gain for the two together to collusively mis-report \( m = l \).

Finally, the last two constraints ensure that both the CEO and expert receive no less than their respective reservation pay-off. Note that the incentive constraint of the CEO, (3), together with the limited liability conditions implies that the CEO receives no less than \( B \), so that the IR constraint of the CEO is never binding.

Before solving the contracting problem where the CEO and the expert are able to perfectly collude, we go to the benchmark case where they cannot collude at all. A comparison between the two cases will pinpoint the source and magnitude of costs incurred by the shareholders due to collusion.

C. The Benchmark: No Collusion

The contracting problem of the case is derived by removing the collusion proofness constraints, namely (6) and (7), with the rest unchanged. What occurs is summarized by the proposition below, where superscript \( \text{'ncx'} \) stands for "non-collusive expert".

**Proposition 1** If expert does not collude with the CEO, then,

(i) the contract to the CEO is: \( w_{hn}^{ncx} = \frac{Bh}{\lambda_\theta (1 - \lambda_\theta) x_h}, w_{ln}^{ncx} = w_{fn}^{ncx} = 0 \), and any contract to the expert is optimal so long as it satisfies (4) and (5) and makes her IR constraint, (8), binding.

(ii) the expected cost of compensation to the CEO is \( C^{ncx} = \frac{B}{\lambda} [q + \frac{(1 - \lambda)x_h}{\lambda_\theta (1 - \lambda_\theta) x_h}] \), and that to the expert is \( B_x \). The gain of the firm from the expert’s service, \( v^{ncx} \), equals \( (\Omega - B_x) + (C^c - C^{ncx}) \), where \( C^c \) is the cost of compensation to the CEO in the absence of the expert, that is, \( C^c = \frac{B}{\lambda} (q + \frac{x_l}{x_h - x_l}) \). And \( C^c > C^{ncx} \).
Result (i) follows the maximum incentive principle, $^5$ which, in the context of our model, means that the CEO is paid only when both the signal is high and the project succeeds. Especially, $w_{i}^{ncx} = 0$, that is, the CEO receives no severance payment.

Result (ii) says that the signal of the non-collusive expert creates value in two channels. First, it is used to improve the interim decision, by which the firm gets the social value of the expert’s service, $\Omega - B_x$. Second, this information, being indicative of the CEO’s effort, is used to reduce the cost of incentivizing the CEO by $C^n - C^{ncx}$, compared to the situation where the firm does not hire the expert.$^6$

In the remainder of this section, we put the collusion back and investigate under what circumstances and in what forms the prospect of collusion may corrode the value of the firm.

D. The Optimal Contracts Under Collusion

The proposition below first discovers the condition under which the collusion loses its bites and is checked by optimal contracting alone. It then goes on to lay down the composition of the CEO compensations and the costs of the firm, when the condition is not honored and thus the collusion bites. Below superscript ‘$x$’ below stands for "collusive expert".

**Proposition 2** (i) If and only if

$$\frac{B/\Delta}{B_x} \leq \frac{(p_h - p_l) [\lambda s_g - (1 - \lambda) s_b]}{p_h p_l} \equiv \psi(\lambda),$$

then the collusion has no impact on the contract to the CEO and on the value of the firm, which are thus the same as in Proposition 1.

(ii) If $B/\Delta > \psi(\lambda)B_x$, then (a): the optimal contract to the CEO and that to the

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$^5$The principle states that the agent shall receive a positive payment only when all the informative signals display the values indicating he has chosen high effort. For exposition of this principle, see Laffont and Martimont (2003) and Bolton and Dwatripont (2005)

$^6$The firm’s problem in this situation is found by deleting all the terms related to the expert in the problem explained in subsection B, that is, $\min_{\{w_h, w_f, w_l\}} e_h w_h + (q_h - e_h) w_f + q_l w_l$ subject to (a), (d) and (f).
expert are:

\[
\begin{align*}
    w_h^x &= \frac{B/\Delta - (2\lambda - 1)(\frac{p_h}{p_t} - p_t)B_x}{A}, \quad w_i^x = \frac{p_t(B/\Delta - \psi(\lambda)B_x)}{A}, \quad w_j^x = 0; \\
x_h^x &= \frac{B_x}{p_h}, \quad x_i^x = B_x, \quad x_j^x = 0,
\end{align*}
\]

where \( \bar{A} \equiv \lambda s_g - (1 - \lambda)s_b - p_t(2\lambda - 1) \). As \( w_i^x > 0 \), the CEO receives a severance payment. And \( w_h^x > w_h^{ncx} \).

(b): The expected cost of compensation to the CEO is

\[
C^x = \frac{(qs_g + (1 - q)s_b)B/\Delta - (\lambda^2 s_g - (1 - \lambda)^2 s_b)(\frac{p_h}{p_t} - p_t)B_x}{A}.
\] (10)

The expected cost of compensation to the expert is \( B_x \). Collusion increases the cost of compensation to the CEO. That is, \( C^x > C^{ncx} \).

We discuss the proposition as follows.

First, condition (9), which differentiates circumstances where the very problem of collusion is relevant from those where it is not, forms a unifying link across the sections of the paper, because similar and parallel conditions will arise in the next two sections. Furthermore, all these conditions display the same two patterns: From (9), the more serious the moral hazard problem of the CEO (measured by a high \( B/\Delta \)), or the less precise the expert’s signal (measured by a low \( \lambda \)),\(^7\) the less likely is condition (9) satisfied and hence the more likely the collusion bites. The intuition behind both patterns is related to the value to the CEO and the expert when the project gets continued. This continuation value is higher, if the moral hazard problem of the CEO is severer because that commands a larger incentive payment \( (w_h) \) to the CEO; or if the informativeness of the expert’s signal \( (\lambda) \) is smaller because that implies a greater probability of success conditional on the bad news \( (p_t) \). A larger continuation value provides the alliance of the CEO and the expert with greater incentives to collusively suppress the bad news and thereby get the project continued.

In particular, the paper observes that the larger the incentive payments to the CEO, the greater the incentives for the CEO and the expert to collude. If the greater incentives to collude lead to higher likelihood of fraudulent activities being exposed,\(^7\) \( \psi(\lambda) = \left( \frac{1}{p_t} - \frac{1}{p_h} \right)[\lambda s_g - (1 - \lambda)s_b] \) increases with \( \lambda \), as \( p_t \) decreases and \( p_h \) increases with it.
then this observation is consistent with the empirical findings that this likelihood is positively linked to the magnitude of the incentive payments, for example, by Agarwal and Chadha (2005) (who link accounting scandals to stock-based compensations); and by Bergstresser and Philippon (2006), Burns and Kedia (2006), and Efendi et. al (2007) (who all link earning misstatements to CEO stockholding).

Second, a clue to why the collusion loses bites when condition (9) is honored is given by result (ii.a), which shows how the firm uses tools of contracting to maximally check the collusion. To encourage the expert to report bad news and discourage her from colluding with the CEO, the firm wants to raise \( x_l \), the payment to her on the report of bad news, which, by (4) and with \( x_f = 0 \), has the upper boundary of \( p_h x_h \) (beyond it she is induced to mis-report good news as bad news). In the optimal contracting, therefore, \( x_l = p_h x_h \). Thus, to suppress the bad news and let the project continue, which then succeeds with probability \( p_l \), the expert loses \( x_l - p_l x_h = (p_h - p_l) x_h \). If the CEO is unable to compensate this loss of her out of his wage \( w_h \), he cannot persuade her to suppress the bad news for his interest. Therefore, the collusion is checked by optimal contracting alone, when \( w_h \) is low relative to \( x_h \), that is, when \( B/\Delta \) is small relative to \( B_x \). So arises condition (9).

Third, when condition (9) is not honored, result (ii.b) shows that solely due to collusion, the cost of compensation to the CEO is increased by a magnitude of \( C^{x_l} - C^{ncx} \). Result (ii.a) establishes that this increment takes two forms, in the severance payment (i.e. \( w_l^x > 0 \)) and in the rise of the incentive payment (i.e. \( w_h^x > w_h^{ncx} \)). The former is to encourage disclosure of bad news. The latter form is because the presence of the severance payment dilutes the incentives of the CEO, whereby the firm has to raise the incentive payment further to him to elicit his effort.

By result (i) of Proposition 1, the collusion between the CEO and the expert could be checked by optimal contracting alone. If it fails, there are still checks and balances both from within and outside the firm. Two of them are examined respectively in the following two sections: (a) The building by the expert of the reputation of being honest and (b) publicly available information.
3 The Internal Checks and Balances: The Reputation Building by the Expert

In this section we consider the building by the expert of the reputation of being honest and non-collusive when she expects a long term relationship with the firm. Specifically, we address the following questions:

1. When does the concern for reputation motivate the expert not to collude with the CEO? How much will the repeated interactions improve over the one-shot interaction in checking collusion problems?

2. In the circumstances where the reputational mechanism works, is it cost free or is it associated with some additional costs to the firm?

To capture the reputational mechanism, in this section we extend the static model outlined in the last section to allow for repeated interactions between the expert and the firm. In order to focus exclusively on the reputational concern of the expert, we abstract the same issue concerned with the CEO, so the contract to the CEO remains static. The discount rate for both the expert and the firm is $\beta \in (0, 1)$.

In each period, which consists of the same three dates, 0, 1, and 2, the firm has a new project, subject to the same type of managerial moral hazard, as was in the preceding section. The novelty of this section is to define two states as to each period: A period is in state 1 if in this period the firm hires the expert; otherwise, it is in state 0.

If the current period is in state 0, the expert gets 0. The project is always continued at date 1 and the firm obtains from the period $(qs_g + (1 - q)s_b)y - C^\alpha$ (recall $C^\alpha$ is the expected payment to the CEO if the firm does not hire the expert). In the next period, it then hires the expert with probability $z_0$, that is, the next period will be in state 1 with probability $z_0$ and will remain in state 0 with probability $1 - z_0$.

Suppose the current period is in state 1, namely the firm is hiring the expert.

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8 A large number of empirical studies show that the relationships between firms and experts (like audit firms, investment bankers etc.) are often long lived; See, Burch, Nanda and Warther (2005), Fang (2000), Rau (2000), Ross (2010) among many others.

9 If expert still gets something by engaging with some other firm, it is even harder for her to establish the reputation and our result is strengthened.
According to the discussion ensuing Proposition 2 above, to curb the collusion problem, the firm pays the expert with \( x_h^* (= \frac{B_h}{p_h}) \) if the project is continued and succeeds, and with \( x_l^* = p_h x_h^* \) if it is liquidated. The compensation contract to the CEO, as shown in the preceding section, depends on whether the expert actually colludes with him or not. The expert receives her private signal, \( m = h \) or \( l \), which is perceived by the CEO. If the true report of the signal reduces the CEO’s expected payoff, he may offer the expert an additional "pay-off" in order to persuade her to submit a report in his favour. Then, the expert’s strategy is to decide whether to report the truth or to make a false report.

The firm’s strategy, on the other hand, consists of a decision, denoted by \( d \), based on the expert’s report on whether to liquidate or to continue the project, and the probabilities in which it will rehire the expert in the next period upon each of the following contingencies: (a) the project is liquidated, (b) it is continued and succeeds, or (c) it is continued and fails; we denote these three probabilities by \( z_l \), \( z_s \), and \( z_f \) respectively. The firm’s strategy in state 1 is thus represented by a profile of \( (d, z_l, z_s, z_f) \).

The state 1 is thus the "reward mode" and the state 0 the “punishment mode” for the expert.\(^{10}\)

While deciding whether to report the truth, the expert faces a trade-off between the current gains from accepting the extra payoffs offered by the CEO versus the loss of future businesses with the firm as the likelihood of her being hired will diminish with lying. If there are equilibria in which the expert, because of the reputational concern, always reports the truth, then it undoes the collusion problem. In the following we first establish that it cannot always undo the collusion problem, but it helps by enlarging the range of parameters where the collusion can be checked. Thereafter, we proceed to the scenario where it is checked by the reputational mechanism.

### A. Infeasibility of Reputation when the Expert’s Signal Is Noisy Enough

To deliver the first point, we seek the necessary conditions under which the expert can establish the reputation of being non-collusive. In the reputational equilibria, given

\(^{10}\)See Myerson (1991) (pages 343-349) on the detailed explanations of these modes.
that the expert does not collude with the CEO, the compensation contract to the CEO specifies \( w^h = w^{nicx}_h \equiv \frac{B/\Delta}{\lambda s_h - (1-\lambda)s_b} \) and \( w_l = 0 \) from Proposition 1. And the project is liquidated when \( \tilde{m} = l \) is reported, and continued when \( \tilde{m} = h \) is reported. Given the firm follows the prescribed rule for liquidation and continuation, we move on to find the conditions for the rest of its strategy, namely, for \( z_0, z_l, z_s, z_f \), under which the incentive compatibility (IC) constraints for the expert to report the truth are satisfied.

Let \( V_1 \) denote the continuation value of the expert at the start of a period in state 1 and let \( V_0 \) denote her continuation value at the start of a period in state 0. Let \( \Pi_1 \) and \( \Pi_0 \) be the counterparts for the firm.

Suppose that the expert has received \( \tilde{m} = h \) in the period. If she reports it truthfully, then the project is continued. With probability \( p_h \), it succeeds, and she is paid with \( x^*_h \) this period and will be rehired with probability \( z_s \) the next period; and with probability \( 1 - p_h \), the project fails and she is paid nothing this period and will be rehired with probability \( z_f \). Therefore, overall, with probability of \( p_h z_s + (1-p_h)z_f \) will she be re-hired, and hence her expected payoff is:

\[
p_h x^*_h + \beta \{ [p_h z_s + (1-p_h)z_f] V_1 + [1 - p_h z_s - (1-p_h)z_f] V_0 \}.
\]

Alternatively, if she reports, dishonestly, that \( \tilde{m} = l \) (which is not in the interest of the CEO), then the project is liquidated and she is paid with \( x_l = p_h x^*_h \) this period and will be re-hired with probability \( z_l \). Therefore, the expert’s expected payoff is:

\[
p_h x^*_h + \beta \{ z_l V_1 + (1-z_l) V_0 \}.
\]

The incentive compatibility (IC) constraint in case of \( \tilde{m} = h \) commands that this payoff is no bigger than that of reporting the truth, or equivalently:

\[
[p_h z_s + (1-p_h)z_f - z_l] [V_1 - V_0] \geq 0. \tag{11}
\]

Now suppose the expert receives \( \tilde{m} = l \). If she reports it truthfully, her expected payoff, as calculated above, is

\[
p_h x^*_h + \beta \{ z_l V_1 + (1-z_l) V_0 \}.
\]

However, in this case, the CEO has interest to distort her report. If the truth, \( \tilde{m} = l \), is reported, the project is liquidated and he gets nothing (\( w_l = 0 \)), while if \( \tilde{m} = h \) is
reported, the project is continued and succeeds with probability \( p \), on which he gets \( w_{hc} \).
Therefore, he is willing to pay any amount of \( T \leq w_{hc} \) in the contingency to buy her to report \( m = h \).\(^{11}\)
If she accepts the bribery and reports \( m = h \), she obtains \( T + x_h^* \) upon the success of the project this period and will be re-hired with probability \( p_l z_s + (1 - p_l) z_f \), because the project is continued and then succeeds with probability \( p_l \) and fails with probability \( 1 - p_l \).
Hence, on mis-reporting, her expected payoff is:
\[
p_l(T + x_h^*) + \beta \{ [p_l z_s + (1 - p_l) z_f] V_1 + [1 - p_l z_s - (1 - p_l) z_f] V_0 \}.
\]
The IC constraint in case of \( m = l \) commands that this payoff is no bigger than that of reporting the truth for any \( T \leq w_{hc} \), or equivalently:
\[
\beta [z_l - p_l z_s - (1 - p_l) z_f] [V_1 - V_0] \geq p_l w_{hc} - (p_h - p_l) x_h^*.
\]
\( (12) \)

There exist reputational equilibria, in which the expert does not collude with the CEO due to the reputational concern, only if the two IC constraints above are satisfied for some \((z_0, z_l, z_s, z_f)\). The proposition below states the condition under which the two IC constrains cannot be satisfied for any \((z_0, z_l, z_s, z_f)\), that is, under which the reputational equilibria do not exist and the collusion problem as discussed in the preceding section remains in force even under repeated interactions.

**Proposition 3** If \( \frac{B/\Delta}{B_x} \geq \psi(\lambda) \frac{1}{1 - p_h} \), with \( \psi(\lambda) \) as defined in \((9)\), then there exist no equilibria where the expert always reports her signal honestly, even with the discount rate \( \beta \) going to 1.

We have three comments on this proposition.

First, the proposition shows that the reputational concerns cannot undo the collusion problem completely. Sometimes, the reputation of being honest cannot be sustained, even if there is no discount for the future benefit.

Second, in the next subsection we show that if the condition of the proposition does not hold, then the reputation can be sustained for \( \beta \) close to 1 enough. That is, the reputational mechanism checks the collusion problem if and only if
\[
\frac{B/\Delta}{B_x} < \psi(\lambda) \cdot \frac{1}{1 - p_h}.
\]
\( (13) \)

\(^{11}\)The bribery may take covert forms as discussed in papers of Demski (2003) and Ritter (2010).
This condition is looser than (9), that under which the collusion can be checked in the static setting: the two inequalities have the same left hand side, but the right hand side of (13) is \( \frac{1}{1-p_h} \) times larger than that of (9). Therefore the reputational concern, though unable to undo the collusion problem completely, helps to check it.

Third, condition (13) displays the same patterns as (9), namely that the more serious the moral hazard problem of the CEO (measured by a high \( B/\Delta \)), or the less precise the expert’s signal (measured by \( \lambda \)),\(^{12}\) the less likely the condition is satisfied and hence the more likely the collusion bites.

We move on to the case where condition (13) holds, and where we show that if \( \beta \) is close to 1 enough, the reputational equilibria exist. We also discuss the properties of the equilibria.

**B. The Reputational Equilibria when the Expert’s Signal Is Sufficiently Informative:**

In this section, we go on to establish main features of the equilibria where the expert maintains the reputation of being non-collusive. We proceed in three steps. First, we show below that there exists a continuum of reputational equilibria in which \( \Pi_1 \) (and \( V_1 \)) is the same but \( \Pi_0 \) (and \( V_0 \)) assumes different values. Next, we focus on the equilibrium in which both \( \Pi_0 \) and \( V_0 \) attain the maximum value. This equilibrium, therefore, is the most efficient one as it Pareto dominates all the other equilibria. Finally, we draw empirical implications of this Pareto dominant equilibrium.

In the truth-telling equilibria, the strategy of the expert is to report truthfully her signal in state 1 and doing nothing in state 0 as she is not hired. And the firm liquidates the project on the report of bad news and continues it on the report of good news. As for the remaining part of the firm’s equilibrium strategy, we apply the “self-generating” approach developed by Abreu, Pearce, and Stachetti (1986) to derive the equilibrium payoff profile \( (\Pi_1, \Pi_0) \).

If the current period is in state 1, so that the expert is hired and she truthfully reports her signal, then this period the firm obtains from the non-collusive expert value

\[ \text{As } \psi(\lambda), \psi(\lambda) \cdot \frac{1}{1-p_h} \text{ increases with } \lambda \text{ as } \frac{1}{1-p_h} \text{ increases with it.} \]
\(v^{ncx}\), which, by Proposition 1 (ii), comes from both the improvement of interim decision \((\Omega - B_x)\) and the reduction in the costs of compensating the CEO \((C^{nx} - C^{ncx})\), and the next period it rehires the expert with probability, as shown above, \(q_hz_h + (1-q_h)z_I\), where \(z_h \equiv p_hz_s + (1-p_h)z_f\) is the rehiring probability when \(\bar{m} = h\) and \(q_h \equiv q\lambda + (1-q)(1-\lambda)\) is the probability of \(\bar{m} = h\). The firm’s present value at the start of a period of state 1 is thus:

\[
v^{ncx} + \beta\{(q_hz_h + (1-q_h)z_I)\Pi_1 + [1 - q_hz_h - (1 - q_h)z_I]\Pi_0\}.
\]

If the current period is in state 0, that is, when the expert is not hired, the firm gets 0 from the expert for this period and rehires her with probability \(z_0\) in the next period. Therefore, its present value is:

\[
\beta[z_0\Pi_1 + (1 - z_0)\Pi_0].
\]

According to Abreu, Pearce, and Stachetti (1986) and Myerson (1991), \((\Pi_1, \Pi_0)\) is a profile of stationary equilibrium payoffs, if and only if

\[
\Pi_1 = \max_{\{z_h, z_s, z_f\}} v^{ncx} + \beta\{(q_hz_h + (1-q_h)z_I)\Pi_1 + [1 - q_hz_h - (1 - q_h)z_I]\Pi_0\}; \tag{14}
\]
subject to the IC constraints, namely (11) and (12), with \(z_h = p_hz_s + (1-p_h)z_f\). And 

\[
\Pi_0 = \max_{z_0} \beta[z_0\Pi_1 + (1 - z_0)\Pi_0], \tag{15}
\]
subject to the same IC constraints.\(^{13}\)

The solution to the above problem is characterized by the following propositions. Let \(\lambda^*\) be the solution to \(\frac{B/\Delta}{B_x} = \psi(\lambda) \cdot \frac{1}{1-p_h}\), then,

**Proposition 4** (a): If \(\lambda > \lambda^*\), namely, if (13) holds, then reputational equilibria exist for \(\beta\) close to 1 enough.

(b): In the unique Pareto dominant equilibrium, the pay-off profile to the firm and that to the expert are as follows:

\[
\begin{align*}
P_1 &= \frac{v^{ncx}}{1 - \beta} [1 - \frac{(1-p_h)T^*}{(p_h - p_I)B_x}], \\
\Pi_0 &= \frac{\beta z_0}{1 - \beta(1-z_0)} \Pi_1; \\
V_1 &= \frac{B_x}{1 - \beta} [1 - \frac{(1-p_h)T^*}{(p_h - p_I)B_x}], \\
V_0 &= \frac{\beta z_0}{1 - \beta(1-z_0)} V_1.
\end{align*}
\]

\(^{13}\)In state 0, the firm does not hire the expert and hence faces no incentive problems in the current period. It still has to honor the same IC constraints, however, because it commits to the same strategy \(z_0\) in all the subsequent periods of state 0 and it knows that this strategy affects the incentives of the expert in the subsequent periods of state 1, as represented by the two IC constraints.
And the rehiring probabilities are:

\[
\begin{align*}
    z_0 &= \min\left(\frac{(p_h - p_l)B_x}{T^*} - (1 - p_h) - \frac{1 - \beta}{\beta}, 1\right); \\
    z_f &= \max(0, 1 - \frac{\beta(p_h - p_l)B_x - \beta(1 - p_h)T^*}{T^*}); \\
    z_l &= p_h + (1 - p_h)z_f; \\
    z_s &= 1,
\end{align*}
\]

where \( T^* \equiv p_lw_{ac}^{\lambda} - (p_h - p_l)x_h^* = \frac{p_l}{\lambda s_g - (1 - \lambda)s_b} \cdot B/\Delta - \frac{p_h - p_l}{p_h} \cdot B_x, \) and at \( \lambda = \lambda^*; \)
\[
1 - \frac{(1-p_h)T^*}{(p_h-p_l)B_x} = 0.
\]

**Proposition 5** (a): All the rehiring probabilities increase with \( \lambda \), that is, \( \frac{\partial z_i}{\partial \lambda} \geq 0 \), for \( i = l, f, 0 \), and \( 1 = z_s > z_l > z_f \).

(b): \( \frac{\partial \Pi}{\partial \lambda} > 0 \) and \( \frac{\partial V_i}{\partial \lambda} > 0 \), for \( i = 1, 0 \).

We infer the following observations from the above propositions.

First, Propositions 3 and 4.a imply that the expert can establish the reputation of being honest if and only if her capability of processing information, measured by \( \lambda \), is above some threshold value of \( \lambda^* \).

Second, \( z_s = 1 \), namely, reporting good news followed by an actual success leads to renewing the business relationship with certainty, or "nothing succeeds like a success".

Third, reporting bad news and reporting good news followed with an actual failure, even though in both cases experts are honest, are punished, in the sense that the business relationship is not renewed with certainty. However, the punishment is smaller for the experts with a higher capability of processing information, namely a larger value of \( \lambda \).

Fourth, the value of the client firms and the expected lifetime income of the expert both increase with \( \lambda \), implying that a more capable expert adds a higher value to its client firms and enjoys a higher expected income.

Lastly, the propositions suggest that using the reputational mechanism to check collusion is not cost free. When having a repeated business relationship with the expert, the firm has two choices. It could either resort to the reputational mechanism to deter the expert from colluding with the CEO, or give it up and pay more to the CEO to elicit
the expert’s information. By the former the firm gets $\Pi_1 = \frac{v^{ncx}}{1-\beta} [1 - \frac{(1-p_h)T^*}{(p_h-p_l)B_x}]$ from the expert, while by the latter it gets $\Pi_1^x = \frac{v^x}{1-\beta}$ where $v^{ncx}$ and $v^x$ are respectively the one-period value of the non-collusive expert and that of the collusive expert as examined in the static model. Intuitively, the advantage of using the reputational mechanism is that for the current period, the firm gets more from the non-collusive expert than from the collusive expert: $v^{ncx} - v^x = C^x - C^{ncx} > 0$ (see Proposition 2). The disadvantage, however, is that the former arrangement commands the firm not to rehire the expert on some contingencies (in order to honour the IC constraints), while the latter does not use the severance of the relationship to provide incentives and thereby the firm gets the advisory service each period; this difference is captured by $1 - \frac{(1-p_h)T^*}{(p_h-p_l)B_x} < 1$. But note that the firm may get it from either the expert of the last period or another one of the same breed, namely, the non-reputable expert is substitutable over time.

When $\lambda \approx \lambda^*$, $1 - \frac{(1-p_h)T^*}{(p_h-p_l)B_x} \approx 0$, and thus $\Pi_1 < \Pi_1^x$, that is, the firm prefer not to use the reputational mechanism, even if it is feasible, for deterring the collusion. This result, together with proposition 3, suggests that only the expert with the top notch of capability (i.e $\lambda$) appears to possess the reputation of being non-collusive in equilibrium: when $\lambda$ is below the threshold $\lambda^*$, the expert is unable to maintain the reputation; and when $\lambda$ is above it, but not by much, the reputation is technically feasible, but not wanted by the firm. If in real life only a small number of expert firms sit within that top notch (which is an empirical question beyond the paper’s scope), then our paper suggests that only this small number of the expert firms are observed to possess the reputation of being honest; in other words, the market of reputable experts is thin. Indeed, we do not observe many highly reputable brand-names in the circle of financial expertise services. For example, we only see four internationally reputed auditing firms; and according to the reputation index for commercial banks constructed by Ross (2010), only three banks occupied top positions from 2000 to 2009; and see Rau (2000) for a similar observation regarding investment bankers and underwriters engaged in M&A.
4 The External Checks and Balances: Public Information

In preceding sections, the only interim information is the expert’s private information. However, usually there is adequate public information about a firm which is beyond the manipulation by the insiders. Media coverage of the firm or its products, analysts opinions and the prices of financial assets issued by the firm, are prominent examples. In this section, we examine the impact of public information on incentives and collusion. We find two channels via which the public information affects the firm’s value. One, it reduces the cost of incentivizing the CEO, because it bears additional information on his effort choice. The other, it has impact on the collusion problem. This impact, surprisingly, is not monotonically favorable.

We retain the basic structure of Section I, but introduce a public signal at $t = 1$, which is observed by everyone, including the CEO and the expert at the time. The time-line is described in figure 2.

Let $\widetilde{s}$ stands for the public signal. It can take either "high" ($h$) or "low" ($l$) value, according to the following conditional distribution:

$$\Pr(\widetilde{s} = h | \text{good state}) = \Pr(\widetilde{s} = l | \text{bad state}) = \mu > 0.5.$$  

By Assumption (1), if both private and public signals are high, then the project should be continued, and if both are low, it should be liquidated. When the signals have opposite values, there are four scenarios as to the socially optimal decision, each being a map from $\{(\tilde{m} = l, \tilde{s} = h), (\tilde{m} = h, \tilde{s} = l)\}$ to $\{$liquidation, continuation$\}$. To abstract away the information value of the public signal and thus to focus on its effects on incentives and collusion, we focus on the scenario where the public signal adds no information to the interim decision, that is, the project should be continued so long as the private signal is high, even if the public signal is low, and it should be liquidated so long as the private signal is low, even if the public signal is high.

Formally, we assume

$$p_{hl}y > M > p_{th}y.$$  

where $p_{hl}$ is the probability of success conditional on the private signal being high and the public signal being low, and $p_{th}$ is the probability of success conditional on the
private signal being low and the public signal being high. (Hereafter, in the subscripts the first term denotes the value of the private signal and the second term that of the public signal.)

As neither the CEO nor the expert shall receive any payment if the project fails, a contract to the CEO is represented by \(w_{ij} = \begin{cases} l;h & \text{if } e_m = l \text{ (so the project is liquidated)} \text{ and } e_s = j \text{ at } t = 1 \text{ and the project succeeds at } t = 2 \end{cases}\). Similarly, a contract to the expert is represented by \(w_{ij} = \begin{cases} l;h & \text{if } e_m = l \text{ (so the project is liquidated)} \text{ and } e_s = j \text{ at } t = 1 \text{ and the project succeeds at } t = 2 \end{cases}\).

Parallel to the analysis in Section I, we present the firm’s problem as designing contracts that minimize the compensation costs subject to the constraints of (a) moral hazard, (b) adverse selection, (c) collusion-proofness, (d) limited liability, and (e) the IR of the expert (as the IR of the CEO is never binding). We present the optimization problem in Appendix C and only the solution of the problem below. Thereafter we discuss its meanings.

**Proposition 6**

(i) If and only if

\[
\frac{B/\Delta}{B_x} \leq \frac{(p_{hh} - p_{lh})(\lambda \mu s_g - (1 - \lambda)(1 - \mu)s_b)}{p_{hh}p_{lh}(q_{hh} + q_{lh})} \equiv \psi(\lambda; \mu),
\]

the collusion problem has no bite on the value of the firm, where \(q_{hh} + q_{lh} = q\mu + (1 - q)(1 - \mu)\) is the probability of \(e_s = h\). Then, the optimal contract to the CEO and an optimal contract to the expert are as follows:

\[
\begin{align*}
\text{w}_{hh} &= \frac{B/\Delta}{\lambda \mu s_g - (1 - \lambda)(1 - \mu)s_b} \quad \text{w}_{hl} = \text{w}_{lh} = \text{w}_{ll} = 0; \\
\text{x}_{hh} &= \frac{B_x}{q_{hh} + q_{lh}p_{hh}}, \quad \text{x}_{lh} = \frac{B_x}{q_{hh} + q_{lh}}, \quad \text{x}_{hl} = \text{x}_{ll} = 0.
\end{align*}
\]

The cost of compensation to the CEO is \(C^{xp} = \frac{B}{\Delta} \left( q + \frac{(1 - \lambda)(1 - \mu)s_b}{\lambda \mu s_g - (1 - \lambda)(1 - \mu)s_b} \right) < C^x\). And \(\psi(\lambda; \mu)\) increases with \(\mu\) if \(\frac{\mu}{1-\mu} \leq \frac{(1-q)s_b}{q s_g}\).

(ii) If condition (17) does not hold, then the optimal contract to the CEO and that to the expert are:

\[
\begin{align*}
\text{w}_{hh} &= \frac{1}{A} \left[ B/\Delta - \frac{(\lambda - \mu)(p_{hh} - p_{lh})}{(q_{hh} + q_{lh}p_{hh})}B_x \right], \quad \text{w}_{lh} = \frac{1}{A} [p_{lh}(B/\Delta - \psi(\lambda; \mu)B_x)], \quad \text{w}_{hl} = \text{w}_{ll} = 0; \\
\text{x}_{hh} &= \frac{B_x}{(q_{hh} + q_{lh})p_{hh}}, \quad \text{x}_{lh} = \frac{B_x}{q_{hh} + q_{lh}}, \quad \text{x}_{hl} = \text{x}_{ll} = 0.
\end{align*}
\]
Here \( A \equiv \lambda \mu s_g - (1 - \lambda)(1 - \mu)s_b - (\lambda - \mu)p_{th} > 0 \). And if \( \frac{\mu}{1 - \mu} \leq \frac{(1 - q)s_b}{q s_g} \), the expected cost of compensation to the CEO \( C^{xp} < C^x \).

The following insights arise out of the proposition.

First, as to the extent to which the collusion problem can be checked, the same patterns as those of the last two sections recur here: the more serious the moral hazard problem of the CEO (measured by \( B = \Delta \)), or the less informative the expert’s signal (measured by \( \lambda \)), the less likely can the collusion be checked, because it will be harder to satisfy the condition (17).

Second, in result (i), that all the \( w_{ij} \) but \( w_{hh} \) equals 0 is driven by the maximum incentive principle, as Proposition 1 (i), which means that the CEO is paid only when the private signal and the outcome of the project, and the public signal, all indicate that he has chosen high effort. In this way, the public signal reduces the cost of incentivizing the CEO. This beneficial effect still presents itself when the collusion problem bites, in the form of \( w_{hl} = 0 \) in result (ii), which means that the negative public signal nullifies the payment to the CEO when \( \bar{m} = h \).

Third, the impact of the public information on the collusion problem takes two forms, one presented in the difference between conditions (17) and (9) under which the collusion has no bite in the respective circumstance, the other in the result \( w_{ll} = 0 \). Let us elaborate on them in order.

When \( \mu = 0.5 \), that is when the public signal is devoid of information, \( \psi(\lambda; \mu) = \psi(\lambda) \) and conditions (17) and (9) coincide. Result (i) shows that \( \psi(\lambda; \mu) > \psi(\lambda) \) at \( \mu \geq 0.5 \). Therefore, when the public signal is not so informative, it helps curb the collusion. When the public signal is very informative (i.e. \( \mu \) is big), however, we can show in the proof of the proposition that \( \psi(\lambda; \mu) \) could fall below \( \psi(\lambda) \), and therefore the public information could exacerbate the collusion problem. An intuition will be clear after we expound the second form the public information takes for its effects on the collusion problem, as below.

In section I we showed that in the absence of the public signal, if the collusion bites, the CEO receives payments so long as the project is liquidated. In the presence of the public signal, by contrast, Proposition 6 (ii) predicts that the CEO receives the severance payment only when the public signal is high, and he receives nothing upon
the liquidation when it is low (i.e. \( w_{ll} = 0 \)), \textit{even though this low public signal does not help induce liquidation}. That is is because the collusion proof constraints are binding only when \( \tilde{s} = h \). This result suggests that collusion is more likely to bite and to arise when the public information pictures a rosy prospect, that is, during booms, which is empirically confirmed by Wang, Winton and Wu (2010).

Having shown that collusion is binding only when \( \tilde{s} = h \), we are now ready to give an intuition for why a strong public signal could exacerbate the collusion problem. When \( \tilde{s} = h \), continuing the project is more profitable to the alliance of the CEO and the expert than it is in the absence of \( \tilde{s} \), because the probability of success \( p_{lh} > p_l \). This force pushes the firm to pay the alliance more to elicit the bad, private news. The second, counter, force, however, is offered by the beneficial effect of the public information in reducing the expected incentive wages. When \( p_{lh} - p_l \) is big enough, that is, when the public signal is sufficiently informative, the first force could overwhelm the counter force, so the public signal could exacerbate the collusion problem, as we remarked above.

\section{5 Conclusion}

Firms extensively use the services of various types of expertise intermediaries, ranging from information disclosures, auditing, to advice on M&As and IPOs. In recent years, the outbreak of scandals involving the expert firms has raised serious concerns regarding their functioning in corporate governance. On this issue, the paper systematically investigates their roles pertaining to information provision, incentives and collusion. The core message of the paper is that the moral hazard problems of the management, or the low capability of the expert firms, not only hurt the client firms individually on its own, but also makes collusion more likely to bite and more difficult to be checked. The paper’s results cover many empirical findings related to fraudulent activities of firms and also put forward some predictions for further tests.

However, our analysis has treated the information alliance as almost a single entity without any friction in between. This approach seems reasonable when the communication between the expert and the CEO is perfect or the expert does not involve hidden
effort in producing the signal. It remains to be seen how our results change once we introduce these elements of friction between the collusive parties. In future, we would like to extend our research along these lines.

6 Appendix

The Proof of Proposition 1:

Noting that the IR constraint to the CEO is never binding and removing the collusion proof constraints, namely (6) and (7), the optimization problem thus becomes:

\[
\min_{\{w_h, w_f, w_l, x_h, x_f, x_l\}} e_h(w_h + x_h) + (q_h - e_h)(w_f + x_f) + q_l(w_l + x_l),
\]

s.t. (3), (4), (5), (8), and the nonnegative constraints. (18)

Note \[\min\{e_h(w_h + x_h) + (q_h - e_h)(w_f + x_f) + q_l(w_l + x_l)\} \geq \min\{e_h w_h + (q_h - e_h)w_f + q_l w_l\} + B x,\] where the second \(\geq\) is because of (8), the IR of the expert. Therefore, we will have solved the optimization problem, if we (a) find a contract to the expert, \(\{x_h, x_f, x_l\}\), that honors all the relevant constraints and makes (8) binding; and (b) find a contract to the CEO, \(\{w_h, w_f, w_l\}\), that attains \(\min\{e_h w_h + (q_h - e_h)w_f + q_l w_l\}\) subject to all the relevant constraints. We can accomplish these tasks when the collusion proof constraints are removed, intuitively, because the removal of them makes the moral hazard problem of the CEO and the adverse selection problem of the expert independent; these two problems intertwine through the collusion proof constraints.

Task (a) is simple. There are actually a continuum of contracts that pass the above standard, for example, \(\{x_h = \frac{B x}{p_h}, x_f = 0, x_l = B x\}\).

For (b): The constraints relevant to \(\{w_h, w_f, w_l\}\) are the nonnegative constraints and (3), which is equivalent to

\[
[\lambda s_g - (1 - \lambda)s_h]w_h + [(\lambda(1 - s_g) - (1 - \lambda)(1 - s_h)]w_f - (2\lambda - 1)w_l \geq B/\Delta. \tag{19}
\]

The task becomes to find a contract \(\{w_h, w_f, w_l\}\) that minimizes the expected payment to the CEO subject to (19) and the nonnegative constraints. The solution is given below.
Claim A: \( w_l = 0, w_f = 0, \) and \( w_h = \frac{B/\Delta}{\lambda s_g - (1 - \lambda) s_b} \).

Proof. First, \( w_l = 0 \): A positive \( w_l \) both worsens (i.e. increases) the objective and tightens constraint (19) because \( 2\lambda - 1 > 0 \).

Second, by the same reason, \( w_f = 0 \) if \( \lambda(1 - s_g) - (1 - \lambda)(1 - s_b) < 0 \). Otherwise, task (b) is equivalent to solving:

\[
\min_{w_h, w_f} \{ e_h w_h + (q_h - e_h) w_f \}, \text{ s.t. } (19) \text{ and } w_h, w_f \geq 0
\]

The solution to the problem is \( w_h = \frac{B/\Delta}{\lambda s_g - (1 - \lambda) s_b} \) and \( w_f = 0 \), because \( \frac{e_h}{\lambda s_g - (1 - \lambda) s_b} < \frac{q_h - e_h}{\lambda(1 - s_g) - (1 - \lambda)(1 - s_b)} \), which is equivalent to: \( e_h \left[ (\lambda(1 - s_g) - (1 - \lambda)(1 - s_b) \right] < \lambda s_g - (1 - \lambda) s_b [q_h - e_h] \Rightarrow \lambda s_g - (1 - \lambda) s_b [q_h - e_h] \Rightarrow (q \lambda s_g + (1 - q)(1 - \lambda) s_b) (2\lambda - 1) < \lambda s_g - (1 - \lambda) s_b \Rightarrow (1 - \lambda) s_b < \lambda s_g \).

Therefore, the solution to task (b) is \( w_h = \frac{B/\Delta}{\lambda s_g - (1 - \lambda) s_b} \) and \( w_f = w_l = 0 \). With these wages, we find the expected payment to the CEO is \( C^{mx} = \frac{B}{\Delta} q + \frac{(1 - \lambda) s_b}{\lambda s_g - (1 - \lambda) s_b} \). And we see the IR constraint to the expert is binding, so the expected payment to her is \( B_x \).

The Proof of Proposition 2:

With the collusion proof constraints, (6) and (7), returned, the optimization problem becomes:

\[
\min_{\{w_h, w_f, w_l, x_h, x_f, x_l\}} e_h(w_h + x_h) + (q_h - e_h)(w_f + x_f) + q_l(w_l + x_l), \quad (20)
\]

s.t. (3) through to (8), and the nonnegative constraints.

For (i): Compared to problem (18) above, two more constraints are added to the current problem, which means the minimum value should weakly increase. Therefore, if we find a contract \( \{w_h, w_f, w_l, x_h, x_f, x_l\} \) that satisfies all the constraints and attain the same minimum value as that of problem (18), that contract must be a solution to the current problem. So consider \( \{w_h = \frac{B/\Delta}{\lambda s_g - (1 - \lambda) s_b}, w_f = w_l = 0\} \), the optimal contract to the CEO derived above, and \( \{x_h = \frac{B_x}{p_h}, x_f = 0, x_l = B_x\} \), an optimal contract to the expert given above. The contract attains the same value of the objective as that of problem (18). It also satisfies all the constraints if \( \frac{B/\Delta}{B_x} \leq \psi(\lambda) \), that is, if condition (9) holds true. Therefore, if the condition holds, that contract is a solution to problem.
(20) and collusion has no impact on the optimal contract to the CEO and the value of the firm.

For (ii): If (9) does not hold, the contract considered in (i), namely \( w_h = \frac{B/\Delta}{\lambda x_h - (1-\lambda)x_f}, w_f = w_l = 0, x_h = \frac{B \times}{p_h}, x_f = 0, x_l = B_x \}, satisfy all the constraints except (7): \( x_l + w_l \geq p_l(x_h + w_h) + (1 - p_l)(x_f + w_f) \). Therefore, this constraint is binding now, from which, furthermore, three claims follow.

First, (6) is non-binding: It, with the binding (7), becomes \( p_h(x_h + w_h) + (1 - p_h)(x_f + w_f) \geq p_l(x_h + w_h) + (1 - p_l)(x_f + w_f) \), which holds because \( p_h > p_l \) and \( x_h + w_h > x_f + w_f \) (the two are paid more when the project succeeds than when it fails).

Second, (4) is binding: To slacken (7), the binding constraint, the firm wants \( x_l \) as large as possible; in economic terms, to encourage disclosure of bad news, the firm wants to pay the expert as much as possible upon the disclosure. Besides (6), which was judged non-binding, the only constraint that up-bounds \( x_l \) is (4), which is therefore binding, namely,

\[
x_l = p_h x_h + (1 - p_h) x_f.
\]  

(21)

Third, \( x_f = w_f = 0 \): With (21) substituted, (7) becomes \( w_l \geq p_l w_h - (p_h - p_l) x_h + (1 - p_l) w_f + (p_h - p_l) x_f \). To slacken it as much as possible, \( x_f = w_f = 0 \). For \( w_f \), moreover, we saw in the proof of Proposition 1 that a positive \( w_f \) increases the cost of incentivizing the CEO.

With \( x_f = w_f = 0 \), (21) becomes

\[
x_l = p_h x_h.
\]  

(22)

Then, the binding (7) becomes

\[
w_l = p_l w_h - (p_h - p_l) x_h.
\]  

(23)

Substitute these two equations and \( w_f = 0 \) into (19), to which (3) is equivalent, and it becomes

\[
\tilde{A} w_h + (2 \lambda - 1)(p_h - p_l) x_h \geq B/\Delta,
\]  

(24)
where $\bar{A} \equiv \lambda s_g - (1 - \lambda)s_b - p_t(2\lambda - 1)$. Substitute (22) and $x_f = 0$ into (8) and note $e_h + q_t p_t = q_h p_h + q_l p_h = p_h$, and (8) then become:

$$p_h x_h \geq B_x. \quad (25)$$

And substitute (22), (23), and $x_f = w_f = 0$ into the objective function of problem (20), and note that $e_h + q_t p_t = e_h + e_l$ and that $e_h + q_t p_h - q_t (p_h - p_t) = e_h + e_l$. The the objective becomes $(e_h + e_l)(w_h + x_h)$.

Therefore, problem (20) becomes:

$$\min_{\{w_h, x_h\}} w_h + x_h, \ s.t. (24), \ (25), \ and \ w_h, x_h \geq 0. \quad (26)$$

Since $\bar{A} > (2\lambda - 1)(p_h - p_t)$ (which is equivalent to $\lambda s_g - (1 - \lambda)s_b > (2\lambda - 1)p_h \iff \lambda s_g - (1 - \lambda)s_b > (2\lambda - 1)\frac{q_h}{q_t} \iff [\lambda s_g - (1 - \lambda)s_b]q_h > (2\lambda - 1)e_h$, which has been shown in the proof of Claim A), the solution to problem (26) is $x_h = \frac{B_x}{q_h}$ and $w_h = \frac{B_x}{\lambda} \cdot [B/\Delta - (2\lambda - 1)\frac{(p_h - p_t)}{p_h} B_x]$, which makes both (24) and (25) binding. Substituting this into (22) finds $x_l$ and into (23) finds $w_l$. All this together with $x_f = w_f = 0$ gives the full optimal contract.

The IR constraint to the expert, to which (25) is equivalent, is binding. Thus the expected payment to the expert is $B_x$. The expected payment to the CEO is to be find by substituting the optimal contract to him and thus equals $C^e = \frac{(q_s + (1-q)s_b)B/\Delta - (\lambda^2 s_g - (1 - \lambda)^2 s_b) \frac{(p_h - p_t)}{p_h} B_x}{\lambda}$. As $C^{ncx} = \frac{B}{\Delta} [q + \frac{(1 - \lambda) s_b}{\lambda s_g - (1 - \lambda)s_b}]$ from Proposition 1, $C^e - C^{ncx} = \frac{\lambda^2 s_g - (1 - \lambda)^2 s_b}{\lambda s_g - (1 - \lambda)s_b} \cdot w^x_I > 0$. Q.E.D.

For the proofs of Propositions 3 to 5, we let $T^* \equiv p_t w_{h}^{ncx} - (p_h - p_t)x_h^*$; and $z_h \equiv p_h z_s + (1 - p_h) z_f$ (the probability of the expert being re-hired if she has truthfully reported $\tilde{m} = h$). With $w_{h}^{ncx}$ given by Proposition 1 and $x_h^* = \frac{B_x}{q_h}$ given by Proposition 2 (ii), $T^* = \frac{B_x}{\lambda s_g - (1 - \lambda)s_b} \cdot B/\Delta - \frac{p_h - p_t}{p_h} \cdot B_x$.

**The Proof of Proposition 3:**

We show that if $\frac{B/\Delta}{B_x} \geq \frac{(p_h - p_t)[\lambda s_g - (1 - \lambda)s_b]}{p_h p_l} \cdot \frac{1}{1 - p_h} = \psi(\lambda) \frac{1}{1 - p_h}$, then there exists no $(z_0, z_l, z_s, z_f)$, all between 0 and 1, that satisfies both (11) and (12). By (11), $z_h \equiv p_h z_s + (1 - p_h) z_f \geq z_l$, which together with (12) implies:

$$\beta(p_h - p_t)(z_s - z_f)(V_1 - V_0) \geq T^*. \quad (27)$$
In a typical setting of Folk Theorem, \( V_1 - V_0 \) is in the order of \( \frac{1}{1-\beta} \) and hence (27) will be satisfied for \( \beta \) close enough to 1. However, in the setting of the paper, since the signal is noisy and hence the expert has to be wrongly punished, we are going to show, \( V_1 - V_0 \) is in the order of \( \frac{1}{1-ph\beta} \), which is upper bounded even at \( \beta = 1 \). For that purpose, we set off to find the following upper bound

**Claim B:** \((z_s - z_f)(V_1 - V_0) \leq \frac{B_x}{1-ph}\) for any \( \beta < 1 \).

**Proof:** Let us go to calculate \( V_1 - V_0 \) for a given \((z_0, z_l, z_s, z_f)\). At a period of state 1, the expert gets \( B_x \) at the period, and she will be rehired next period with probability \( qhz_h + (1 - qh)z_l \) (recall \( qh \equiv q\lambda + (1-q)(1-\lambda) \) is the probability of \( \hat{m} = h \)). Therefore,

\[
V_1 = B_x + \beta \{ [qhz_h + (1 - qh)z_l]V_1 + [1 - qhz_h - (1 - qh)z_l]V_0 \}.
\]

We saw that (11) implies \( z_h \geq z_l \). Thus, \( qhz_h + (1 - qh)z_l \leq z_h \). Then, \( V_1 \leq B_x + \beta [z_h V_1 + (1 - z_h) V_0] \Rightarrow V_1 \leq \frac{B_x + \beta (1 - z_h) V_0}{1 - \beta z_h} \). It follows that \( V_1 - V_0 \leq \frac{B_x - (1 - \beta) V_0}{1 - \beta z_h} < \frac{B_x}{1 - \beta z_h} \), where \( V_0 \geq 0 \) because of limited liability.

Therefore, \((z_s - z_f)(V_1 - V_0) < B_x \cdot \frac{z_s - z_f}{1 - z_h} = B_x \cdot \frac{z_s - z_f}{1 - ph z_s - (1-ph) z_f} \equiv B_x \cdot f(z_s, z_f)\). Note that \( f \) increases with \( z_s \), which is no bigger than 1. Hence, \( f(z_s, z_f) \leq f(1, z_f) = \frac{1}{1-ph} \).

So is the claim proved. q.e.d.

By the claim, the right hand side of (27) is smaller than \( \frac{ph - p_l}{1-ph} B_x \) for any \( \beta \). Therefore, if \( \frac{ph - p_l}{1-ph} B_x \leq T^* \Leftrightarrow \frac{ph - p_l}{1-ph} B_x \leq \frac{p_l}{\lambda s_l - (1-\lambda) s_s} \cdot B/\Delta \cdot \frac{ph - p_l}{p_l} \cdot B_x \Leftrightarrow \psi(\lambda) \frac{1}{1-ph} \leq \frac{B/\Delta}{B_x} \), no \((z_0, z_l, z_s, z_f)\) satisfies (27), to which the IC constraint (12) is equivalent, even with \( \beta \) going to 1. Q.E.D.

**The Proof of Proposition 4:**

With the approach of self generating, \((\Pi_1, \Pi_0)\) is a profile of equilibrium strategies if and only if

\[
\Pi_1 = \max_{(z_l, z_s, z_f)} v^{ncx} + \beta [Q\Pi_1 + (1 - Q)\Pi_0], \text { s.t. (11), (12)}; \tag{29}
\]

\[
\Pi_0 = \max_{z_0} \beta [z_0\Pi_1 + (1 - z_0)\Pi_0], \text { s.t. (11), (12)}, \tag{30}
\]

where \( Q(z_l, z_s, z_f) \equiv qhp_h z_s + qh(1-p_h)z_f + (1-qh)z_l \), that is, \( Q \) denotes the probability of the expert being rehired next period if she is hired this period. Constraints (11) and
We proceed the following steps: first we find out $V_1 - V_0$ as a function of $(z_0, z_l, z_s, z_f)$, then solve the maximization problem for $\Pi_1$ and that for $\Pi_0$, and finally characterize the Pareto dominant equilibrium.

**Step 1: $V_1 - V_0$ as a function of $(z_0, z_l, z_s, z_f)$**.

The equation for $V_1$ has been given by (28) above, replicated below,

$$V_1 = B_x + \beta [Q V_1 + (1 - Q) V_0].$$  \hfill (33)

If she is not hired this period, she gets 0 now and will be rehired with probability $z_0$. Therefore,

$$V_0 = \beta [z_0 V_1 + (1 - z_0) V_0].$$  \hfill (34)

From these two equations,

$$V_1 - V_0 = \frac{B_x}{1 - \beta Q + \beta z_0}.$$

Substitute it into (32), which then becomes

$$\beta (z_l - p_l z_s - (1 - p_l) z_f) \frac{B_x}{1 - \beta Q + \beta z_0} \geq T^*.$$  \hfill (35)

**Step 2: Solve problem (14)**.

The problem is equivalent to maximizing $Q$, the rehiring probability, subject to the same constraints. With (11) equivalent to (31), (12) to (32) and then to (35), the problem is:

$$\max_{\{x_l, x_s, x_f\}} Q(z_l, z_s, z_f), \text{s.t. (31) and (35)}.$$  

To solve the problem, first note that $\frac{\partial Q}{\partial z_l} > 0$. Thus, an increment in $z_l$ both improve the objective function and slackens (35), which means $z_l$ should be increased until (31) is binding. Hence,

$$z_l = z_h = p_h z_s + (1 - p_h) z_f.$$

Then, $Q = p_h z_s + (1 - p_h) z_f$ and (35) becomes:

$$\beta (p_h - p_l) B_x \frac{z_s - z_f}{1 + \beta z_0 - \beta p_h z_s - \beta (1 - p_h) z_f} \geq T^*.$$  \hfill (36)
An increment in \( z_s \) both improves the objective as \( \frac{\partial Q}{\partial z_s} > 0 \) and slackens (36), and therefore should be carried out to its maximum. That is, \( z_s = 1 \).

By substituting \( z_s = 1 \) into the objective function and (36), we see problem (14) now becomes:

\[
\max_{\{z_0, z_f\}} Q = p_h + (1 - p_h)z_f,
\]

\[
s.t. \beta(p_h - p_l)B_x \frac{1 - z_f}{1 - \beta + \beta z_0 + \beta(1 - p_h)(1 - z_f)} \geq T^*.
\] (37)

The objective function goes up with an increase in \( z_f \); which tightens the constraint. Therefore, the constraint is binding, which gives:

\[
\frac{1 - z_f}{1 - \beta + \beta z_0} = \frac{T^*}{\beta(p_h - p_l)B_x - \beta T^*(1 - p_h)}.
\] (38)

This equation defines \( z_f \) as a function of \( z_0 \), denoted by \( z_f = h(z_0) \).

To sum up, the solution to problem (14), namely, the equilibrium strategy of the firm in state 1, is:

\[
z_s = 1, z_f = h(z_0), z_l = p_h + (1 - p_h)z_f.
\] (39)

Step 3: Solve problem (15).

Following the analysis above, it is equivalent to:

\[
\max_{\{z_0, z_f\}} z_0, \ s.t.(37).
\]

Again, the constraint is binding, which gives the same equation, (38), whereby \( z_0 \) is a function of \( z_f \), that is, \( z_0 = h^{-1}(z_f) \), where

\[
h^{-1}(z_f) = (1 - z_f)[\frac{(p_h - p_l)B_x}{T^*} - (1 - p_h)] - \frac{1 - \beta}{\beta}.
\]

That is, any combination of \( z_f \) and \( z_0 \) forms an equilibrium so long as it satisfies (38), namely, \( z_f = h(z_0) \) or \( z_0 = h^{-1}(z_f) \).

Therefore, the reputational equilibria exist, if and only if there exists a profile of \((z_0, z_l, z_s, z_f)\), of which the elements are all between 0 and 1 and satisfy (39).

Note that such a profile exists if and only if (13) holds, which proves Proposition 4(a): \( h^{-1}(z_f) \) is maximized at \( z_f = 0 \) and the maximal equals \( \frac{(p_h - p_l)B_x}{T^*} - (1 - p_h) - \frac{1 - \beta}{\beta} \).

If and only if (13) holds, then \( \frac{(p_h - p_l)B_x}{T^*} - (1 - p_h) > 0 \), and the maximal \( h^{-1}(0) \geq 0 \) for \( \beta \) close to 1 enough.
On the other hand, if (13) holds, there is a continuum of \((z_0, z_l, z_s, z_f)\) of which the elements are all between 0 and 1 and satisfy (39), that is, a continuum of reputational equilibria.

Step 4: The Pareto dominant equilibrium.

Substitute the equilibrium strategy, (39), into the formula for \(\Pi_1, \Pi_0, V_1,\) and \(V_0,\) namely (29), (30), (33), and (34). And we find

\[
\Pi_1 = \frac{v^{ncx}}{1 - \beta} \left[ 1 - \frac{(1 - p_h)T^*}{(p_h - p_l)B_x} \right] \quad \text{and} \quad \Pi_0 = \frac{\beta z_0}{1 - \beta (1 - z_0)} \Pi_1; \quad (40)
\]

\[
V_1 = \frac{B_x}{1 - \beta} \left[ 1 - \frac{(1 - p_h)T^*}{(p_h - p_l)B_x} \right] \quad \text{and} \quad V_0 = \frac{\beta z_0}{1 - \beta (1 - z_0)} V_1. \quad (41)
\]

It follows that in all the reputational equilibria, \(\Pi_1\) and \(V_1\) are the same, but \(\Pi_0\) and \(V_0\) both increases with \(z_0.\) Therefore, the equilibrium with the highest \(z_0\) Pareto dominates all the others and is characterized below.

\[
z_0 = h^{-1}(z_f) = (1 - z_f) \left[ \frac{(p_h - p_l)B_x}{T^*} - (1 - p_h) \right] - \frac{1 - \beta}{\beta}. \quad \text{The function } z_0 = h^{-1}(z_f) \text{ is maximized at } z_f = 0 \text{ and the maximal equals } \frac{(p_h - p_l)B_x}{T^*} - (1 - p_h) - \frac{1 - \beta}{\beta}. \text{ If this maximal is no bigger than 1, it gives the highest possible } z_0. \text{ On the other hand, if the expression exceeds 1, the highest } z_0 = 1 \Rightarrow z_f = 1 - \frac{T^*}{\beta(p_h - p_l)B_x - \beta(1 - p_h)T^*}. \quad \text{Therefore, in the unique Pareto dominant equilibrium, } (\Pi_1, \Pi_0) \text{ is given by (40) and the rehiring probabilities are:}
\]

\[
\begin{align*}
z_0 & = \min \left( \frac{(p_h - p_l)B_x}{T^*} - (1 - p_h) - \frac{1 - \beta}{\beta}, 1 \right); \\
z_f & = \max (0, 1 - \frac{T^*}{\beta(p_h - p_l)B_x - \beta(1 - p_h)T^*}); \\
z_l & = p_h + (1 - p_h)z_f; \\
z_s & = 1.
\end{align*}
\]

It is easy to check \(1 - \frac{(1 - p_h)T^*}{(p_h - p_l)B_x} = 0 \iff \frac{p_h - p_l}{1 - p_h} B_x = T^* \iff \psi(\lambda) \frac{1}{1 - p_h} = B/\Delta B_x \iff \lambda = \lambda^*.
\]

The Proof of Proposition 5:

Note that \(T^* = \frac{p_l}{\lambda s_g - (1 - \lambda)s_b} \cdot B/\Delta - \frac{v_h - v_l}{p_h} \cdot B_x\) decreases with \(\lambda,\) because \(\frac{p_l}{\lambda s_g - (1 - \lambda)s_b}\) decreases with it and \(\frac{v_h - v_l}{p_h}\) increases with it. Then, a direct differentiation of those variables \((\Pi_1, \Pi_0, V_1, V_0)\) and \((z_l, z_f)\) with respect to \(\lambda\) proves the proposition.

The Proof of Proposition 6:
To prove the proposition, first we describe the optimization problem. Let $q_{ij}$ for $i, j = h, l$ denote the ex ante probability of the event $\tilde{m} = i, \tilde{s} = j$. Thus,

\[
\begin{align*}
q_{hh} &= q\lambda\mu + (1 - q)(1 - \lambda)(1 - \mu) \\
q_{hl} &= q(1 - \mu) + (1 - q)(1 - \lambda)\mu \\
q_{lh} &= q(1 - \lambda)\mu + (1 - q)\lambda(1 - \mu) \\
q_{ll} &= q(1 - \lambda)(1 - \mu) + (1 - q)\lambda\mu
\end{align*}
\]

And let $e_{ij}$ for $i, j = h, l$ denote the ex ante probability of the event $\tilde{m} = i, \tilde{s} = j$ and the project succeeds if it is continued at $t = 1$. Then

\[
\begin{align*}
e_{hh} &= q\lambda s_g + (1 - q)(1 - \lambda)(1 - \mu)s_b \\
e_{hl} &= q\lambda(1 - \mu)s_g + (1 - q)(1 - \lambda)\mu s_b \\
e_{lh} &= q(1 - \lambda)\mu s_g + (1 - q)\lambda(1 - \mu)s_b \\
e_{ll} &= q(1 - \lambda)(1 - \mu)s_g + (1 - q)\lambda s_b
\end{align*}
\]

We have

\[p_{ij} = \frac{e_{ij}}{q_{ij}} \text{ for } i, j = h, l.\]

With these notations, the problem is:

\[
\min_{\{w_{ij}, x_{ij} | i = h, l; j = h, l\}} e_{hh}(w_{hh} + x_{hh}) + e_{hl}(w_{hl} + x_{hl}) + q_{lh}(w_{lh} + x_{lh}) + q_{ll}(w_{ll} + x_{ll}),
\]

subject to:

(a) Moral hazard constraint of the CEO, which commands that he get more if exerting effort than if shirking:

\[
[\lambda s_g - (1 - \lambda)(1 - \mu)s_b]w_{hh} + [\lambda(1 - \mu)s_g - (1 - \lambda)\mu s_b]w_{hl} - (\lambda - \mu)w_{lh} - (\lambda + \mu - 1)w_{ll} \geq \frac{B}{\Delta} \tag{42}
\]

(b) Adverse selection constraint of the expert, which commands that the expert alone does not gain from mis-reporting her private signal, conditional on any observed
public signal:

\[ p_{hh} x_{hh} \geq x_{lh} \quad (43) \]
\[ p_{hl} x_{hl} \geq x_{ll} \quad (44) \]
\[ x_{lh} \geq p_{hh} x_{hh} \quad (45) \]
\[ x_{ll} \geq p_{hl} x_{hl} \quad (46) \]

(c) Collusion proof constraints, which commands that the expert and the CEO jointly do not gain from mis-reporting the private signal, conditional on any observed public signal:

\[ p_{hh}(x_{hh} + w_{hh}) \geq x_{lh} + w_{lh} \quad (47) \]
\[ p_{hl}(x_{hl} + w_{hl}) \geq x_{ll} + w_{ll} \quad (48) \]
\[ x_{lh} + w_{lh} \geq p_{hh}(x_{hh} + w_{hh}) \quad (49) \]
\[ x_{ll} + w_{ll} \geq p_{hl}(x_{hl} + w_{hl}) \quad (50) \]

(d) Limited liability constraints: \( w_{ij} \geq 0, x_{ij} \geq 0 \), for \( i = h, l \) and \( j = h, l \).

(e) The IR to the expert:

\[ e_{hh} x_{hh} + e_{hl} x_{hl} + q_{hh} x_{lh} + q_{ll} x_{ll} \geq B_x \quad (51) \]

Then Proposition 6 is proved as follows.

For (i): As we saw in the proof of Proposition 2(i), for the firm, the best thing is to (a) use both the public information and the expert’s information to incentivize the CEO and meanwhile (b) to pay \( B_x \) to the expert, but (c) is not worried about the collusion problem. The contract to the CEO and that to the expert given by the proposition implement both (a) and (b): the contract to the CEO follows the maximum incentive principle and thus gives the maximum incentives to the CEO; and the contract to the expert gives her \( B_x \) only. Furthermore, with these contracts, the collusion proof constraints, (47) through to (50), are all satisfied, if

\[ \frac{p_{hh}}{\lambda\mu s_y - (1-\lambda)(1-\mu)s_b} \cdot \frac{B}{\lambda} = -\frac{p_{hh} - p_{hl}}{(q_{hh} + q_{hl})p_{hh}}. \]

\( B_x \leq 0 \), namely if condition (17) holds true. Hence if condition (17) holds true, the collusion problem has no bites and the contracts given by the proposition form a solution to the optimization problem of the firm.

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The cost of compensation to the CEO, \( C^{xp} \), is to be found by substituting the values of the wages: \( C^{xp} = \frac{B}{\Delta} \left( q + \frac{(1-\lambda)(1-\mu)s_h}{\mu_s g - (1-\lambda)(1-\mu)s_h} \right) \). That compensation in the absence of the public signal, \( C^x \), at its lowest, is \( C^{nx} = \frac{B}{\Delta} \left[ q + \frac{(1-\lambda)s_h}{\mu_s g - (1-\lambda)s_h} \right] \) given by Proposition 1. It is straightforward to see \( C^{xp} < C^{nx} \). Therefore, \( C^{xp} < C^x \).

The right hand side of the condition, \( \psi(\lambda; \mu) \), increases with \( \mu \) if \( \mu \leq \frac{(1-q)s_h}{q s_g} \). We have \( \frac{\lambda \mu s_g - (1-\lambda)(1-\mu)s_h}{q u + (1-q)(1-\mu)} \bigg|_{\mu = \frac{(1-q)s_h}{q s_g}} = \frac{\lambda \mu s_g - (1-\lambda)(1-\mu)s_h}{q t + (1-q)} \) increases with \( t \) and the derivative goes to 0 when \( t \to \infty \) (i.e. \( \mu \to 1 \)). And \( \frac{p_h - p_h}{p_h P_h} = \frac{q_{lh} e_{hh} - q_{hh} e_{lh}}{e_{hh}} \). We find

\[
q_{lh} e_{hh} - q_{hh} e_{lh} = q(1-q)\mu(1-\mu)(2\lambda - 1)(s_g - s_b).
\]

and therefore, \( \frac{p_h - p_h}{p_h P_h} = \left[ q(1-q)(2\lambda-1)(s_g - s_b) \right] \frac{\mu(1-\mu)}{[q \lambda \mu s_g + (1-q)(1-\lambda)(1-\mu)s_h][q(1-\lambda)\mu s_g + (1-q)\lambda(1-\mu)s_h]} \).

And

\[
\left[ q \lambda \mu s_g + (1-q)(1-\lambda)(1-\mu)s_h \right] \frac{\mu(1-\mu)}{[q \lambda \mu s_g + (1-q)(1-\lambda)(1-\mu)s_h][q(1-\lambda)\mu s_g + (1-q)\lambda(1-\mu)s_h]} \bigg|_{\mu = \frac{(1-q)s_h}{q s_g}} = \left[ [q \lambda s_g + (1-q)(1-\lambda)s_b] \frac{1}{t} \right] \frac{\mu(1-\mu)}{[q \lambda s_g + (1-q)(1-\lambda)s_b][q(1-\lambda)s_g t + (1-q)\lambda s_b]} .
\]

We have \( \left[ [q \lambda s_g + (1-q)(1-\lambda)s_b] \frac{1}{t} \right] \frac{\mu(1-\mu)}{[q \lambda s_g + (1-q)(1-\lambda)s_b][q(1-\lambda)s_g t + (1-q)\lambda s_b]} \bigg|_{\mu = \frac{(1-q)s_h}{q s_g}} \leq 0 \Leftrightarrow t \leq \frac{(1-q)s_h}{q s_g} \). Furthermore, the derivative of this part does not go to 0 even if \( t \to \infty \).

\( \psi(\lambda; \mu) \) could decreases with \( \mu \) if \( \mu \to 1 \) and thus \( t \to \infty \): The analysis above shows that when \( t \to \infty \), \( \psi(\lambda; \mu) \) is determined by \( \left( \frac{p_h - p_h}{p_h P_h} \right)' \) which is negative if \( t > \frac{(1-q)s_h}{q s_g} \).

For (ii): If condition (17) fails to hold, then (49) is violated, which means it is now binding. Thus

\[
w_{lh} = p_h w_{hh} - (x_{lh} - p_{lh} x_{hh}) . \tag{52}
\]

Furthermore, As was in the case without the public signal, the firm would like to pay the expert as much as possible upon the report of \( \tilde{m} = l \), which implies (43) and (44) are binding:

\[
x_{lh} = p_h x_{hh} \tag{53}
\]

\[
x_{ll} = p_{hl} x_{hl} \tag{54}
\]

Substitute them into (51) and note that \( e_{hh} = q_{hh} p_{hh} \) and \( e_{hl} = q_{hl} p_{hl} \), which then becomes:

\[
(q_{hh} + q_{lh})p_{hh} x_{hh} + (q_{hl} + q_{ll})p_{hl} x_{hl} \geq B_x . \tag{55}
\]

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Substitute (53) into (52), which then becomes:

\[ w_{lh} = p_{lh}w_{hh} - (p_{hh} - p_{lh})x_{hh}. \]  

(56)

Substitute (54) into (50), which then becomes:

\[ w_{ll} \geq p_{ll}w_{ll} - (p_{hl} - p_{ll})x_{hl}. \]  

(57)

Lastly, \( w_{hl} = 0 \) at the optimum, because it follows the maximum incentive principle and furthermore, a positive \( w_{hl} \) tightens (57), the collusion proof constraint in case of \( \tilde{m} = l \) and \( \tilde{s} = l \), thus making the collusion more likely bite.

Substitute \( w_{hl} = 0 \) and (56) into (42), and note

\[ A \cdot s_{gb}(1-\lambda) \cdot s_{gb} = (1-\lambda)(1-\mu) \cdot s_{gb} - (\lambda - \mu)p_{ph}. \]  

The constraint then becomes:

\[ A w_{hh} + (\lambda - \mu)(p_{hh} - p_{lh})x_{hh} - (\lambda + \mu - 1)w_{ll} \geq \frac{B}{\Delta}. \]  

(58)

Substitute \( w_{hl} = 0 \), (56), (53), and (54) into the objective, which becomes

\[ (e_{hh} + e_{lh})(w_{hh} + x_{hh}) + (q_{hl} + q_{ll})p_{hl}x_{hl} + q_{ll}w_{ll}. \]

The optimization problem thus becomes:

\[
\begin{aligned}
\min_{w_{hh}, w_{ll}, x_{hh}, x_{hl}} & \quad (e_{hh} + e_{lh})(w_{hh} + x_{hh}) + (q_{hl} + q_{ll})p_{hl}x_{hl} + q_{ll}w_{ll}, \\
\text{s.t.} & \quad (58), (55) \text{ and } w_{hh}, w_{ll}, x_{hh}, x_{hl} \geq 0.
\end{aligned}
\]  

(59)

To solve this problem, we make three observations.

First note \( w_{ll} = 0 \) at the optimum: A positive \( w_{ll} \) only tightens (58).

Second, (55) is binding, because \( A > (\lambda - \mu)(p_{hh} - p_{lh}) \), which is equivalent to

\[ \lambda \mu s_g - (1 - \lambda)(1 - \mu)s_b - (\lambda - \mu)p_{lh} > (\lambda - \mu)(p_{hh} - p_{lh}) \iff \lambda(1 - \mu)s_g - (1 - \lambda)\mu s_b > (\lambda - \mu)p_{ph} \iff q_{hh}(\lambda(1 - \mu)s_g - (1 - \lambda)\mu s_b) > (\lambda - \mu)e_{hh}, \]  

while

\[ q_{hh}(\lambda(1 - \mu)s_g - (1 - \lambda)\mu s_b) - (\lambda - \mu)e_{hh} = [q_{hh}(\lambda(1 - \mu)s_g - (1 - \lambda)\mu s_b)](\lambda(1 - \mu)s_g - (1 - \lambda)\mu s_b) - (\lambda - \mu)[q_{hh}(\lambda(1 - \mu)s_g + (1 - \mu)(1 - \mu)(1 - \mu)s_b] = \lambda(1 - \lambda)(q_{hh} + q_{ll})p_{hl}x_{hl} > 0. \]

Therefore, in constraint (58), the coefficient of \( w_{hh} \) is larger than that of \( x_{hh} \), but they have the same coefficient in the objective function. It follows that the firm wants \( x_{hh} \) as small as possible, until (55) is binding. From the binding constraint, we have

\[ (q_{hl} + q_{ll})p_{hl}x_{hl} = B_x - (q_{hh} + q_{lh})p_{hh}x_{hh}. \]  

(60)
Substitute it into the objective function, the coefficient of $x_{hh}$ is then $e_{hh} + e_{lh} - (q_{hh} + q_{lh})p_{hh}\big{|}_{e_{hh}=q_{hh},p_{hh}} = e_{lh} - q_{lh}p_{hh} = q_{lh}p_{lh} - q_{lh}p_{hh} = -(p_{hh} - p_{lh})q_{lh} < 0$.

Third, (58) is binding; otherwise, $w_{hl}$ could be reduced to improve the objective value. With $w_{ll} = 0$, the binding (58) is:

$$Aw_{hh} + (\lambda - \mu)(p_{hh} - p_{lh})x_{hh} = \frac{B}{\Delta}$$  \hspace{1cm} (61)

Based on these three observations and with $w_{ll} = 0$ and (60) substituted into the objective function, problem (59) becomes:

$$\min_{w_{hh}, w_{ll}, x_{hh}, x_{hl}} (e_{hh} + e_{lh})w_{hh} - (p_{hh} - p_{lh})q_{lh}x_{hh}, \hspace{0.5cm} s.t. \hspace{0.5cm} (61)$$

$$B_x - (q_{hh} + q_{lh})p_{hh}x_{hh} \geq 0,$$ \hspace{1cm} (62)

where constraint (62) is due to (60) and the nonnegative constraint $x_{hl} \geq 0$.

At the optimum, $x_{hh}$ is made as big as possible and therefore constraint (62) is binding, which, together with (60), gives

$$x_{hh} = \frac{B_x}{(q_{hh} + q_{lh})p_{hh}}; x_{hl} = 0.$$  

Substituting this into (53) and (54) finds $x_{lh}$ and $x_{ll}$.

Substituting $x_{hh} = \frac{B_x}{(q_{hh} + q_{lh})p_{hh}}$ into (61) finds

$$w_{hh} = A^{-1}[B/\Delta - (\lambda - \mu)(p_{hh} - p_{lh})B_x].$$

Putting this and $x_{hh} = \frac{B_x}{(q_{hh} + q_{lh})p_{hh}}$ into (56) finds $w_{lh}$ and we already know $w_{hl} = w_{ll} = 0$. So have we found the whole optimal contract to the CEO and that to the expert, \{w_{ij}, x_{ij}|i, j = h, l\}.

Again, the cost of compensation to the CEO, $C^{xp}$, is to be found by substituting the values of the wages: $C^{xp} = A^{-1}[(q\mu s_g + (1 - q)(1 - \mu)s_b)B/\Delta - \frac{p_{hh} - p_{lh}}{(q_{hh} + q_{lh})p_{hh}}\mu(1 - \mu)(\lambda^2 s_g - (1 - \lambda)^2 s_b)B_x].$ And when $\frac{\mu}{1 - \mu} < \frac{(1 - q)s_b}{q s_g}$, we saw in (i) that $\psi(\lambda; \mu) > \psi(\lambda).$ Therefore, if $\frac{B/\Delta}{B_x} > \psi(\lambda; \mu)$, then $\frac{B/\Delta}{B_x} > \psi(\lambda)$; namely, if collusion bites with the public signal, it bits without it. That is, $C^{xp}$ is to be compared to the value of $C^x$ given by (10) in Proposition 2(ii). Then, $C^{xp} < C^x$. It is equivalent to

$$\frac{1}{A}[(q\mu s_g + (1 - q)(1 - \mu)s_b)B/\Delta - \frac{p_{hh} - p_{lh}}{(q_{hh} + q_{lh})p_{hh}}\mu(1 - \mu)(\lambda^2 s_g - (1 - \lambda)^2 s_b)B_x]$$

$$< \frac{1}{A}[(q s_g + (1 - q)s_b)B/\Delta - (\lambda^2 s_g - (1 - \lambda)^2 s_b)\frac{(p_{hh} - p_{lh})}{p_{lh}}B_x],$$  \hspace{1cm} (63)
where \( A = \lambda \mu s_g - (1 - \lambda)(1 - \mu)s_b - (\lambda - \mu)p_{th} \) and \( \tilde{A} = \lambda s_g - (1 - \lambda)s_b - (2\lambda - 1)p_l \).

To prove it, first, we show

\[ A > \tilde{A} \mu. \]

\[ \Leftrightarrow \lambda \mu s_g - (1 - \lambda)(1 - \mu)s_b - (\lambda - \mu)p_{th} \land (\lambda - \lambda)s_b - (2\lambda - 1)p_l) \mu \Leftrightarrow (1 - \lambda)s_b(2\mu - 1) + (2\lambda - 1)\mu p_l > (\lambda - \mu)p_{th} \Leftrightarrow (2\lambda - 1)\mu p_l \Leftrightarrow (2\lambda - 1)\mu q_l (\lambda - \lambda)q_l(1 - \lambda)s_g + (1 - q)\lambda s_b(1 - \lambda)(1 - \mu)s_b), \text{which follows from } (2\lambda - 1)\mu q_l(\lambda - \mu)q_l \mu \land (2\lambda - 1)\mu q_l(1 - \mu), \text{and the latter is implied by the former because } \mu > 1 - \mu; \text{therefore, it follows from } (2\lambda - 1)\mu q_l(\lambda - \mu)q_l \mu \Leftrightarrow (2\lambda - 1)(1 - \lambda)q_l(1 - \mu) > (\lambda - \mu)(q(1 - \lambda) + (1 - q)\lambda) \Leftrightarrow (\lambda - 1)(2\mu - 1) > 0, \text{true.}

Since \( A > \tilde{A} \mu \), (63) follows from \((q\mu s_g + (1 - q)(1 - \mu)s_b)B/\Delta - \frac{p_{hh} - p_{th}}{(q_{hh} + q_{th})p_{hh}}(1 - \mu)(\lambda^2 s_g - (1 - \lambda)^2 s_b)Bx < \mu[(q\mu s_g + (1 - q)s_b)B/\Delta - (\lambda^2 s_g - (1 - \lambda)^2 s_b)(p_{hh} - p_{th})/p_{hh}] \Leftrightarrow (\lambda^2 s_g - (1 - \lambda)^2 s_b)Bx \mu \frac{(p_{hh} - p_{th})/p_{hh}}{(q_{hh} + q_{th})p_{hh}(1 - \mu)} < (2\mu - 1)(1 - q)s_b)B/\Delta. \text{Note that } \frac{p_{hh} - p_{th}}{p_{hh}} = 1 - \frac{p_{hh}}{p_{hh}} > 1 - \frac{p_{hh}}{p_{hh}} \text{because } \frac{p_{hh}}{p_{hh}} < \frac{p_{hh}}{p_{hh}} \text{(namely, the effect of a high public signal on negating the low private signal is bigger than its effect on enhancing the high private signal). And } 1 - \frac{1 - \mu}{q_{hh} + q_{th}} = 1 - \frac{1 - \mu}{q_{hh} + (1 - q)(1 - \mu)} = \frac{(2\mu - 1)q}{q_{hh} + (1 - q)(1 - \mu)}.

Therefore, the term within ” \{\}\” is smaller than \( \frac{p_{hh} - p_{th}}{q_{hh} + q_{th}}(2\mu - 1)q \). Then the inequality at the end of the chain follows from \( (\lambda^2 s_g - (1 - \lambda)^2 s_b)Bx \frac{q_{mu}}{q_{mu} + (1 - q)(1 - \mu)} \frac{p_{hh} - p_{th}}{p_{hh}} < (1 - q)s_b)B/\Delta \Leftrightarrow \frac{B/\Delta}{Bx} > \frac{\lambda^2 s_g - (1 - \lambda)^2 s_b}{q_{mu} + (1 - q)(1 - \mu)} \frac{p_{hh} - p_{th}}{p_{hh}} \Leftrightarrow \frac{\mu}{\lambda^2 s_g - (1 - \lambda)^2 s_b} < \frac{1 - \mu}{q_{mu} + (1 - q)(1 - \mu)}p_{hh}, \text{This is implied by } (17) \text{if } \frac{\mu}{\lambda^2 s_g - (1 - \lambda)^2 s_b} < \frac{1 - \mu}{q_{mu} + (1 - q)(1 - \mu)} \Leftrightarrow \lambda^2 s_g - (1 - \lambda)^2 s_b < \frac{1 - \mu}{q_{mu} + (1 - q)(1 - \mu)} < \frac{1 - \mu}{q_{mu} + (1 - q)(1 - \mu)} \Leftrightarrow \lambda^2 s_g - (1 - \lambda)^2 s_b < \frac{\lambda^2 s_g - (1 - \lambda)(1 - \mu)s_b}{1 - \mu} \Leftrightarrow \lambda^2 s_g - (1 - \lambda)^2 s_b < \lambda s_g - (1 - \lambda)s_b, \text{true.} \]
Figure 1: Timing without the public signal

Figure 2: Timing with the public signal
References


