The imprecision of volatility indexes

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     Example: When examining the effect of macroeconomic shocks (Bloom, 2009).

CBOE introduced VIX futures in 2004 and options in 2006. In 2012, open interest for futures at 326,066 and options at 6.3 million contracts.
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VIX is imprecise!

Example: Vega VIX

In our sample, the size of the 95% confidence band for Vega VIX (VVIX) is 2.9 percentage points in the median case.

Concern about imprecision in a VIX estimator arises due to aggregation of imprecise implied volatilities (IVs). Latane and Rendleman, 1976; Hentschel, 2003; Jiang and Tian, 2007
Consequences of imprecision

1. Imprecise option prices.
   - Example: A 6100 OTM call option on the Nifty index is priced at Rs.1.92 when using a VVIX of 17.82%.
     (Underlying=5464.75; Maturity=29 days)

   The 95% confidence interval (CI) for VVIX ranges from 16.03% to 19.91% ⇒ the option’s price may lie between Rs.0.89 and Rs.3.86.

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- Estimate the imprecision of model based VIXs.
- Use bootstrapping to estimate the imprecision in a VIX estimator.
- Compute $\sigma$ and confidence bands to measure this imprecision.
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  VVIX estimate - 21.53%
  95% CI - [20.8, 22.32]
- Similarly, for Nifty options with 29 and 57 days to expiry:
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Imprecision indicators are used for model selection: vega, liquidity, and elasticity weighted VIXs. VVIX has the lowest imprecision with a median CI width of 2.9pp.
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The imprecision of volatility indexes
The imprecision of volatility indexes

Outline

- Concerns about measurement
- Measuring the imprecision in a VIX
- Two empirical examples
- Using this measure of imprecision for model selection
- Imprecision of VIX as a measure of ambiguity
- Conclusion
Concerns about measurement
Two approaches to measurement

- Model based approach - uses option pricing model - VXO, VEGA VVIX etc.
  - Measurement errors in prices - imprecise IVs (Hentschel, 2003)
  - Hentschel (2003) derives CI’s from B-S formula.
  - For an ATM stock option with 20 days to expiry, the 95% CIs are of the order +/- 6 pp.
  - For VXO, the 95% CIs are of the order +/- 25 bps.
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- Model free approach - pricing of variance swap - CBOE VIX
  - Methodological errors (Jiang and Tian, 2005)
  - Imprecise intra-day VIX due to varying strike range (Andersen et al., 2011)
Measuring the imprecision in a volatility index
Our approach to the problem

- Non-parametric methodology; contrast with Hentschel (2003).
- Model based; contrast with model free.
- Agnostic about the distribution of errors.
- Each option price is an imprecise transformation of the true implied volatility index.
- Bootstrapping to estimate the imprecision in the VIX estimator.
An example: Vega weighted VIX

The VVIX is computed from all option prices as follows:

1. Estimation of IVs using the Black-Scholes model for the two nearest maturities.

The expected volatility, VVIX, is interpolated to compute the 30 day expected volatility.
An example: Vega weighted VIX

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2. Computation of the average weighted IV for each maturity \( i \):

\[
IV_i = \frac{\sum_{j=1}^{n} w_{ij} IV_{ij}}{\sum_{j=1}^{n} w_{ij}}
\]

where, \( IV_{ij} \) refers to a vector of IVs for \( j = \{1 \ldots n\} \) and two nearest maturities, \( i = \{\text{near}, \text{next}\} \), \( w_{ij} \) refers to the vega weight for the corresponding \( IV_{ij} \).
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3. The vega weighted average IVs are interpolated to compute the 30 day expected volatility, VVIX.
Bootstrap inference: The case of LIBOR

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- The parallel with LIBOR suggests a bootstrap inference approach for VVIX.
Steps involved

1. At time $t$, we observe a chain of option prices for:
   - different strikes
   - two nearest maturities
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4. Each of these datasets summarised into a vega weighted average IV.

5. The vega weighted IVs are interpolated to obtain the VVIX estimate.

6. Repeat steps 3 - 5 $R$ times – bootstrap distribution of the statistic.

7. Now, compute:
   - Standard deviation ($\sigma$)
   - Confidence bands – adjusted bootstrap percentile method (Efron, 1987)
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Data description

- S & P 500 index (SPX) options end-of-day data.
- The data is available for the months of Sep, Oct, and Nov 2010.
- Nifty options tick-by-tick data (~ 200K obs. per day):
- The data is available from Feb, 2009 to Sep, 2010.
- Each dataset includes:
  - Transaction date
  - Expiry date of the options contract
  - Strike price
  - Type of the option i.e. call or put
  - Price of the underlying index
  - Best buy price and ask price of option
- The one and three month MIBOR rates provided by NSE as the riskfree rates.
- The one and three month US Treasury bill rates provided by the US department of the Treasury as the riskfree rates.
Sampling procedure

- We follow Andersen et al. (2011) and sample options as follows:
  1. Construct fifteen seconds series for each individual option using the previous tick method from tick-by-tick data.
  2. Retain the last available quotes prior to the end of each fifteen second interval throughout the trading day.
  3. If no new quote arrives in a fifteen second interval, the last available quote prior to the interval is retained.
  4. If no quote is available in the previous interval, the last available quote from the last five minutes is retained.
  5. Filter out options with zero traded volume (optional).

- For robustness check, sampling frequencies of thirty and sixty seconds are also used.
Two empirical examples
Intuition

We use a sample of near-the-money SPX options.

The underlying is at 1125.59, the number of days to expiry is 29 and the risk-free rate is 0.12%.

<table>
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<th>Strike</th>
<th>Type</th>
<th>Mid-Quote</th>
<th>IVol (%)</th>
<th>Strike</th>
<th>Type</th>
<th>Mid-Quote</th>
<th>IVol (%)</th>
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</table>

Note: We define near-the-money-options as call and put options with strike-to-spot ratio between 0.97 and 1.03 (Pan and Poteshman, 2006).

95% CI of sample mean: [17.65, 18.84]
## The imprecision of volatility indexes

A sample of SPX options

<table>
<thead>
<tr>
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<th>Mid-Quote</th>
<th>Maturity</th>
<th>Risk-free</th>
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## The imprecision of volatility indexes

A single replicate

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<th>Mid-Quote</th>
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</table>
The one-day change in VVIX is smaller than 1.5pp on 62% of the days.
The distribution of VVIX on 2010-09-01: Nifty

The one-day change in VVIX is smaller than 4pp on 92% of the days.
Imprecision of VVIX over a large sample of Nifty options

- The imprecision indicators are computed from Feb 2009 to Sep 2010.
- The median CI for VVIX is 2.9pp which is an economically significant one.
- This is larger than the one-day change in VVIX of 1.18pp.
Using this measure of imprecision for model selection
Benchmarking performance of VIXs

- Alternatives to Vega: elasticity, liquidity etc. (Grover and Thomas, 2012).
Benchamarking performance of VIXs

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- Precision is desirable in an estimator.
- Smaller $\sigma$ and confidence interval $\Rightarrow$ higher precision.
Methodology

- Competitors:
  - Vega weighted VIX: VVIX
  - Liquidity weighted VIX: SVIX, TVVIX
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- Period of analysis: February 2009 - September 2010. Four snapshots a day.

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- Significant test: Pair wise Wilcoxon signed rank test.

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### Summary statistics

<table>
<thead>
<tr>
<th></th>
<th>SVIX</th>
<th>TVVIX</th>
<th>VVIX</th>
<th>EVIX</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Size of confidence band (pp)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min</td>
<td>0.929</td>
<td>1.362</td>
<td>1.033</td>
<td>2.177</td>
</tr>
<tr>
<td>1st Qu</td>
<td>2.713</td>
<td>2.743</td>
<td>2.271</td>
<td>6.201</td>
</tr>
<tr>
<td>Median</td>
<td>3.546</td>
<td>3.418</td>
<td>2.923</td>
<td>7.368</td>
</tr>
<tr>
<td>Mean</td>
<td>4.542</td>
<td>4.024</td>
<td>3.907</td>
<td>8.245</td>
</tr>
<tr>
<td>3rd Qu</td>
<td>4.845</td>
<td>4.440</td>
<td>4.064</td>
<td>9.262</td>
</tr>
<tr>
<td>Max</td>
<td>52.940</td>
<td>23.790</td>
<td>50.490</td>
<td>51.080</td>
</tr>
<tr>
<td>Std Dev</td>
<td>3.803</td>
<td>2.109</td>
<td>3.636</td>
<td>3.926</td>
</tr>
</tbody>
</table>

|                      |      |       |      |      |
| **σ of the bootstrap estimates (pp)** |      |       |      |      |
| Min                  | 0.239| 0.344 | 0.255| 0.571|
| 1st Qu               | 0.706| 0.706 | 0.581| 1.576|
| Median               | 0.913| 0.877 | 0.739| 1.868|
| Mean                 | 1.139| 1.028 | 0.945| 2.053|
| 3rd Qu               | 1.252| 1.141 | 1.025| 2.324|
| Std Dev              | 0.822| 0.530 | 0.754| 0.875|
### The imprecision of volatility indexes

#### Pairwise comparisons: Wilcoxon sign rank test

<table>
<thead>
<tr>
<th></th>
<th>Size of confidence band</th>
<th>( \sigma ) of the bootstrap estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median Diff</td>
<td>Pval</td>
</tr>
<tr>
<td>EVIX - SVIX</td>
<td>3.745</td>
<td>0.000</td>
</tr>
<tr>
<td>EVIX - TVVIX</td>
<td>3.846</td>
<td>0.000</td>
</tr>
<tr>
<td>EVIX - VVIX</td>
<td>4.326</td>
<td>0.000</td>
</tr>
<tr>
<td>SVIX - TVVIX</td>
<td>-0.004</td>
<td>1.000</td>
</tr>
<tr>
<td>SVIX - VVIX</td>
<td>0.641</td>
<td>0.000</td>
</tr>
<tr>
<td>TVVIX - VVIX</td>
<td>0.618</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Ranking:** VVIX, SVIX & TVVIX, EVIX
Imprecision of VIX as a measure of ambiguity

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Improvement of VIX as a measure of ambiguity

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- The proposed measure of imprecision of VIX, might prove to be useful in quantifying the extent of ambiguity that is present at a point in time.
Reproducible research

R package *ifrogs* has been released into the public domain, with an open source implementation of the methods of this paper.
Conclusions

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Imprecision in \( \text{VIX} \) is significant – estimated from SPX or Nifty options. Use the imprecision indicators for model selection. \( \text{VVIX} \) is the most precise estimator.

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Thank you