Other Reduced-form Models

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Credit Risk Elective
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Towards a default-risk adjusted discounting…

• Example: if risk neutral probability of default of a zero-coupon bond promising $1 at maturity $T$ is $p(T)$ and if recovery in default is zero, then risk-neutral expected payoff at $T$ is $(1-p(T))$ and current price is:

$$e^{-r(T-t)}[(1-p(T)) \times 1 + p(T) \times 0] = e^{-r(T-t)} (1 - p(T))$$

• If we substitute for $p(T)$ we obtain first “hint” of useful trick and a link between term structure and intensity based models:

$$e^{-r(T-t)} (1 - p(T)) = e^{-r(T-t)} e^{-\lambda(T-t)} = e^{-(r+\lambda)(T-t)}$$

• In other words price is just face value ($1) discounted at the default risk adjusted rate of $(r+\lambda)$ and the yield spread is just $\lambda$.

However, this result depends strongly on recovery assumptions.
Recovery of Market Value (RMV)

• Suppose that in default investors receive a constant fraction, \( R \), of pre-default value of the defaultable bond
• Default time = \( \tau \), Value of bond instant before default = \( V(\tau-) \)
• Value of bond in default = \( R \cdot V(\tau-) \), \( R = \text{RMV fraction} \)
• Under this assumption we can show that the current value is simply the promised value discounted at the default-adjusted rate \( r + (1-R)\lambda \):

\[ e^{-(r+(1-R)\lambda)(T-t)} \]

• Why is this valuation formula a neat analytical result?
  ✓ Risk-less claims: Discount promised CFs at risk-free rate
  ✓ Risky claims: Discount promised CFs at default risk-adjusted rate
• First, we provide an informal proof. Next, we apply it.
Informal Proof of RMV Result

• Suppose we are valuing a bond at time $t$, that $\lambda$ is the risk neutral default intensity and $R$ the recovery rate, then at time $t+1$ the investor receives:

$$
V_{t+1} \quad \text{if no default with RN prob } (1 - \lambda \Delta t) \\
RV_{t+1} \quad \text{if default with RN prob } \lambda \Delta t
$$

• The price at $t$ is the risk neutral expected payoff discounted at $r$:

$$
V_t = \frac{V_{t+1} (1 - \lambda \Delta t) + V_{t+1} R \lambda \Delta t}{1 + r \Delta t} = \frac{V_{t+1}}{1 + \hat{r} \Delta t}
$$

• Solving and then letting $\Delta t$ tend to zero gives:

$$
\hat{r} = \frac{r + (1 - R) \lambda}{1 - \lambda (1 - R) \Delta t} \quad \text{and as } \Delta t \to 0, \quad \hat{r} \to r + (1 - R) \lambda
$$
Implementing Intensity Models with Recovery of Market Value (RMV) as Default-Adjusted Short Rate Tree
Binomial Model with RMV Recovery: Duffie-Singleton Model – 2 Period Example – RMV Recovery

• Assume: risk-free short rate process (default-free yield curve):

\[ r = 7\% \]
\[ r = 9\% \]
\[ r = 13\% \]

• assume:
  ✓ recovery rate \( R = 0.3 \)
  ✓ annual (risk-neutral) default probability \( \lambda = 0.05 \)

• In practice: use prices of credit risky bonds to fit default intensity
Duffie-Singleton Valuation of 2-Period Bond without default-adjusted rates

\[ V_{13} = \frac{0.95 \cdot 100 + 0.05 \cdot 30}{1.13} = 85.398 \]

\[ V_9 = \frac{0.95 \cdot 100 + 0.05 \cdot 30}{1.09} = 88.532 \]

\[ r = 7\% \]

- no default: payoff = 85.398, prob. = 0.95
- default: payoff = 0.3*85.398, prob. = 0.05

\[ E(\text{Payoff}) = 0.95 \times 85.398 + 0.05 \times 0.3 \times 85.398 = 82.409 \]

\[ V_7 = \frac{0.5 + 82.409}{1.07} = 78.431 \]

\[ r = 9\% \]

- no default: payoff = 88.532, prob. = 0.95
- default: payoff = 0.3*88.532, prob. = 0.05

\[ E(\text{Payoff}) = 0.95 \times 88.532 + 0.05 \times 0.3 \times 88.532 = 85.433 \]

\[ V_7 = \frac{0.5 + 85.433}{1.07} \]
Assuming RMV we can rewrite the calculations in terms of a default-adjusted rate:

\[
\hat{r} = \frac{r + \lambda (1 - \Delta)R}{-\lambda (1 - \Delta)R} \lambda \Rightarrow R. \Delta (year)
\]

<table>
<thead>
<tr>
<th>riskless rate</th>
<th>default adjusted rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>7%</td>
<td>10.881%</td>
</tr>
<tr>
<td>9%</td>
<td>12.953%</td>
</tr>
<tr>
<td>13%</td>
<td>17.098%</td>
</tr>
</tbody>
</table>
Duffie-Singleton Valuation of 2-Period Bond with default-adjusted rates

\[ \hat{r} = 17.098\% \]

\[ V_{13} = \frac{100}{1.17098} = 85.398 \]

\[ \hat{r} = 12.903\% \]

\[ V_7 = \frac{0.5 \times 85.398 + 0.5 \times 88.532}{1.10881} = 78.431 \]

\[ V_9 = \frac{100}{1.1295} = 88.532 \]
Risk-neutral versus Actual Default Probabilities
Estimated 1-year default probabilities for Vintage Petroleum.

Source: Berndt, Douglas, Duffie, Ferguson and Schranz, “Measuring Default Risk Premia from Default Swap Rates and EDFs”, 2004
CDS Rates (approx. equal to spread) and Natural Default Probabilities

Source: Berndt, Douglas, Duffie, Ferguson and Schranz, “Measuring Default Risk Premia from Default Swap Rates and EDFs”, 2004
What is going on?

• It is possible that there are large risk premia associated with default.

• But is also possible that credit spreads are influenced by other factors such as
  ✓ Limited liquidity of corporate debt
  ✓ Institutional limitations on arbitrage between debt and equity

• It turns out that for some derivatives this will make little difference but for others it will be important
  ✓ GM and Ford downgrades of 2004-05
Do Recovery Rate Assumptions Matter?
How much difference do the recovery assumptions make?

• Recovery of market value leads to very convenient valuation formulae but may (or may not) be empirically realistic.
• How much difference does this assumption make?
• The jury is still out, but in many cases, the recovery assumption choice is second order to getting the likelihood of default right
Figure 2: For fixed ten-year par-coupon spreads, $S$, this figure shows the dependence of the mean hazard rate $\bar{h}$ on the assumed fractional recovery $1 - \bar{L}$. The solid (dashed) lines correspond to the model $RFV$ ($RMV$).

Source: Duffie & Singleton