Credit Risk
Introduction

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Credit Risk Elective
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Credit Risk: the Main Issues

• Understanding what determines the value and risk characteristics of instruments which are sensitive to default risk ("defaultable")
  ✓ corporate debt
  ✓ (some) sovereign debt
  ✓ most OTC derivatives (even derivatives on government debt)
  ✓ credit derivatives (even if default free)

• Why is this a “hot” topic?
  ✓ risk management and regulatory rules: need to include credit risks
  ✓ inefficiencies in pricing credit … profitable opportunities?
  ✓ “Boom” and “bust” of credit derivatives
    ➢ the 2007-09 (??) credit crunch – ouch! 😞
  ✓ Future of credit derivatives and their regulation
Size of the Credit Derivatives Market

Source: British Bankers Association; ISDA (2005 and after) – 2007: to mid-year
Outstanding Credit Default Swaps (CDS) in Notional ($billions), Source: ISDA
CDO Issuance ($millions), Source: SIFMA
SIZE of Mortgage-backed CDO Issuance ($mm),
Source: SIFMA
Thriving banks may be needed for credit derivatives to thrive again... How soon will that be?
Course objectives

• Significant progress in credit risk modelling over last few years
• All recent models use “option pricing” technology but two main strands have emerged:
  ✓ structural models: look like option pricing models in which defaultable bond is contingent claim on assets of firm
  ✓ reduced-form or “default intensity” models: these models look more like term structure models
• Objectives: to understand
  ✓ nature of default risk
  ✓ alternative approaches to valuing credit risky instruments
  ✓ credit derivatives: credit default swaps (CDS) and collateralised debt obligations (CDO)
  ✓ liquidity effects NOT captured in current models and how some of these contributed to the ongoing crisis…
Getting Started: the Price of a “Defaultable” Zero Coupon Bond

• The payoff on a defaultable zero-coupon bond is

   Face Value - "Loss in Default"

• We can therefore think of the value of a defaultable zero-coupon bond as:

   \[ PV(\text{Face Value}) - PV(\text{"Loss in Default"}) = \text{Price(Riskless Zero)} - \frac{1}{1 + R^F} \left( R^N - \text{Prob(default)} \times \text{Loss in Default} \right) \]

   Credit risk price discount
Key Elements in Valuing the Credit Risk Discount

\[
\frac{1}{1 + R} \left( \text{RN-Prob(default)} \times \text{Loss in Default} \right)
\]

Credit risk price discount

- **Event** of default
  - ✓ economic mechanism
  - ✓ definition – particularly important in credit derivative contracts
- **Probability** of default?
- **Loss rate** (or recovery) in default
- **Riskless** term structure
- **Risky** term structure or the dynamics of credit risk
This lecture

• Traditional approaches to assessment of credit risk
  ✓ credit ratings
    ➢ One-period
    ➢ Multi-period
  ✓ Z-score

• Determinants of credit spreads – some very simple intuition
  ✓ default rates
  ✓ recovery rates
  ✓ cyclicality of default rates and recovery rates

• Conceptual introduction to structural and reduced-form models
Traditional Approaches to Assessment of Credit Risk

The Rating System
Credit Rating Systems

- Traditional approach to assessment of credit risk employs credit ratings
- These use accounting data, historical default frequencies, judgmental factors etc.

<table>
<thead>
<tr>
<th>Description</th>
<th>Moody’s</th>
<th>S&amp;P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highest safety</td>
<td>Aaa</td>
<td>AAA</td>
</tr>
<tr>
<td>High quality</td>
<td>Aa1, Aa2, Aa3</td>
<td>AA+, AA, AA-</td>
</tr>
<tr>
<td>Upper medium</td>
<td>A1, A2, A3</td>
<td>A+, A, A-</td>
</tr>
<tr>
<td>Lower medium</td>
<td>Baa1, Baa2, Baa3</td>
<td>BBB+, BBB, BBB-</td>
</tr>
<tr>
<td>Junk Bonds</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low grade</td>
<td>Ba1, Ba2, Ba3</td>
<td>BB+, BB, BB-</td>
</tr>
<tr>
<td>Highly spec</td>
<td>B1, B2, B3</td>
<td>B+, B, B-</td>
</tr>
</tbody>
</table>
One Year, Weighted-Average Default Rates by Rating

Source: Moody’s
• What counts as \textit{default}?
Moody's Definition Of Default: Three Types of Credit Event

1. A missed or delayed disbursement of interest and/or principal, including delayed payments made within a grace period;
2. Bankruptcy, administration, legal receivership, or other legal blocks (perhaps by regulators) to the timely payment of interest and/or principal;
3. A distressed exchange occurs where: (i) the issuer offers bondholders a new security or package of securities that amount to a diminished financial obligation (such as preferred or common stock, or debt with a lower coupon or par amount, lower seniority, or longer maturity); or (ii) the exchange had the apparent purpose of helping the borrower avoid default.

• The definition of a default is intended to capture events that change the relationship between the bondholder and bond issuer from the relationship which was originally contracted, and which subjects the bondholder to an economic loss.
• Technical defaults (covenant violations, etc.) are not included in Moody's definition of default.
What do credit ratings mean?

- Credit ratings are intended partly, but not purely, as measures of *default probability*
- Credit ratings are stable measures of credit quality *through the business cycle*
  - Lehman Brothers’ rating was A2 until 15 September 2008 when it filed for bankruptcy, and then B3!
- Rating also reflects “quality of assets” and likely loss in event of default (*loss-given-default* or *LGD*)
- Rating *momentum*: Bad news appears not to be incorporated in one step: a downgrade is more likely to be followed by another downgrade than an upgrade.
Rating Momentum: Three-Year Default Rates Conditional on Last Rating Change within 12 Months

Source: Moody’s, “Rating Transitions and Defaults Conditional on Watchlist, Outlook and Rating History”, 2004
One-Year and Multiple Year Default Rates: *Transition Matrices*
5, 10, 15, and 20-Year Default Rates, 1920-97

Source: Moody’s
Relation between one-year and multi-year default rates: transition matrices

• If one-year probability of default of a bond with current rating A is \( p_A \) then probability of survival is \( (1 - p_A) \)
• But, default probability over two years is not

\[
1 - (1 - p_A)^2
\]

because bond rating at end of year may be different and probability of default in second year higher or lower

• The effect of rating migration can be modelled using transition matrices, the probability that a bond with a rating of A, say, at the start of the year is A, AA, AAA, BBB etc. at the end
Multi-year transition matrices

- E.g. assume “High” state and “Low” state of credit quality with a one period transition matrix

\[
M = H \begin{bmatrix} H & L \\ L & H \end{bmatrix}
\]

- We want to find the two-period matrix: e.g., suppose current rating is A -- rating can remain A or migrate to B:

\[
\begin{align*}
M^2 &= (H \cdot 0.9 + L \cdot 0.1)(H \cdot 0.9 + L \cdot 0.1) \\
&= 0.83 \\
M^2 &= (H \cdot 0.8 + L \cdot 0.2)(H \cdot 0.8 + L \cdot 0.2) \\
&= 0.17
\end{align*}
\]
Multi-year transition matrices (cont’d)

• Two-period transition matrix is just square of one period transition matrix

\[
M^2 = \begin{bmatrix} .9 & .1 \\ .2 & .8 \end{bmatrix} \begin{bmatrix} .9 & .1 \\ .2 & .8 \end{bmatrix} = \begin{bmatrix} .9^2 + (.2)(.1) & (.9)(.1) + (.1)(.8) \\ (.2)(.9) + (.8)(.2) & (.2)(.1) + .8^2 \end{bmatrix} = \begin{bmatrix} .83 & .17 \\ .34 & .66 \end{bmatrix}
\]

• \(n\)-period transition matrix is just \(M^n\)
## Credit Transition Matrix: Example

<table>
<thead>
<tr>
<th>Initial Rating</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>90.81</td>
<td>8.33</td>
<td>0.68</td>
<td>0.06</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AA</td>
<td>0.70</td>
<td><strong>90.65</strong></td>
<td>7.79</td>
<td>0.64</td>
<td>0.06</td>
<td>0.14</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.09</td>
<td>2.27</td>
<td><strong>91.05</strong></td>
<td>5.52</td>
<td>0.74</td>
<td>0.26</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>BBB</td>
<td>0.02</td>
<td>0.33</td>
<td>5.95</td>
<td><strong>86.93</strong></td>
<td>5.30</td>
<td>1.17</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>BB</td>
<td>0.03</td>
<td>0.14</td>
<td>0.67</td>
<td>7.73</td>
<td><strong>80.53</strong></td>
<td>8.84</td>
<td>1.00</td>
<td>1.06</td>
</tr>
<tr>
<td>B</td>
<td>0.11</td>
<td>0.24</td>
<td>0.43</td>
<td>6.48</td>
<td><strong>83.46</strong></td>
<td>4.07</td>
<td>5.20</td>
<td></td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.22</td>
<td>1.30</td>
<td>2.38</td>
<td>11.14</td>
<td><strong>64.86</strong></td>
<td>19.79</td>
<td></td>
</tr>
<tr>
<td>Default</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

Source: Moody’s
Transition Matrices and Default Rates

- Multiplying the one-year transition matrix by itself once, twice, three times etc., gives the default probabilities over 2, 3, 4 years etc.

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>0.00</td>
<td>0.00</td>
<td>0.09</td>
<td>0.43</td>
<td>2.18</td>
<td>5.98</td>
<td>20.46</td>
</tr>
<tr>
<td>10 years</td>
<td>2.34</td>
<td>2.72</td>
<td>3.87</td>
<td>7.26</td>
<td>27.71</td>
<td>38.59</td>
<td>53.79</td>
</tr>
</tbody>
</table>

Source: Standard & Poor’s CreditReview
Z-scores
Accounting-based Statistical Models

• **Z-score** by Altman (1968, 2000) and its variants
  
• The Z-score is given by:

\[
Z = 0.012 X_1 + 0.014 X_2 + 0.033 X_3 + 0.006 X_4 + 0.999 X_5
\]

• **Contributors to the Z-score:**
  
✓ \( X_1 = \) Net Working Capital / Total Assets in % (e.g., 15 for 15%)
  
✓ \( X_2 = \) Retained Earnings / Total Assets in %
  
✓ \( X_3 = \) EBIT / Total Assets in %
  
✓ \( X_4 = \) Market Value of Common and Preferred Stock / Book Value of Debt in %
  
✓ \( X_5 = \) Sales / Total Assets expressed as a ratio (e.g., 1.5)
Z-score

• The higher the Z-score, the lower the probability of bankruptcy

• How is the Z-score calculated?
  ✓ *multiple discriminant analysis*
    - linear function of accounting variables to discriminate between the two groups: defaulting vs. non-defaulting
    - minimize variance within groups while maximizing variance across groups

• The model predicts bankruptcy two years ahead with good accuracy and five years ahead with some accuracy
Bankrupt and non-bankrupt firms

• Altman found the following values for bankrupt and non-bankrupt set of firms:

<table>
<thead>
<tr>
<th></th>
<th>BANKRUPT</th>
<th>NON-BANKRUPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net WC</td>
<td>-6.1%</td>
<td>41.4%</td>
</tr>
<tr>
<td>Retained earnings</td>
<td>-62.6%</td>
<td>35.5%</td>
</tr>
<tr>
<td>EBIT</td>
<td>-31.8%</td>
<td>15.4%</td>
</tr>
<tr>
<td>Market Value</td>
<td>40.1%</td>
<td>247.7%</td>
</tr>
<tr>
<td>Sales</td>
<td>1.5%</td>
<td>1.9%</td>
</tr>
</tbody>
</table>

• Based on these, Z-score for bankrupt firms = -0.25 and that for non-bankrupt firms = +4.88

• Z-score thresholds: Bankrupt Zone-of-ignorance Non-bankrupt
  
  <1.81  1.81-2.99 >2.99
## WorldCom Z-score

<table>
<thead>
<tr>
<th>Ratio</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working capital / Total Assets</td>
<td>0.08</td>
<td>0.08</td>
<td>0.00</td>
</tr>
<tr>
<td>Retained Earnings / Total Assets</td>
<td>0.01</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>EBIT / Total Assets</td>
<td>0.08</td>
<td>0.08</td>
<td>0.02</td>
</tr>
<tr>
<td>Market value of equity / Book value of total liabilities</td>
<td>3.58</td>
<td>1.13</td>
<td>0.54</td>
</tr>
<tr>
<td>Net Sales / Total Assets</td>
<td>0.39</td>
<td>0.40</td>
<td>0.34</td>
</tr>
<tr>
<td><strong>Z-score</strong></td>
<td>2.697</td>
<td>1.274</td>
<td>0.798</td>
</tr>
</tbody>
</table>
Z-score predicted Enron default better than stock markets due to greater reliance on balance-sheet data.
Understanding the spread – first steps
What Determines the Credit Spread? – A Very Simple Model

• Suppose a one-period defaultable bond pays
  ✓ 100: if no default (prob. = 1−p)
  ✓ 100(1−L): if default (prob. = p)

where L is the percentage loss-given-default (LGD)

• Equating 100 discounted at “promised” yield to present value of expected payoff (using riskless rate to discount):

\[
\text{spread} \quad y \quad R \quad Lp
\]

\[
\frac{100}{1 + y} = \frac{100(1−p) + 100(1−L)p}{1 + R}
\]

i.e., spread is equal to \( Lp \), the “expected loss rate”

• if \( L = 1 \), i.e., if LGD is 100%, then – ignoring risk premia – spread is equal to default probability
### Average Credit Spreads and Default Probabilities

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Average Default Probability 10 yrs</th>
<th>Average Default Probability 1 year</th>
<th>Average Credit Spread 10 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>0.77%</td>
<td>0.0077%</td>
<td>0.63</td>
</tr>
<tr>
<td>Aa</td>
<td>0.99%</td>
<td>0.099%</td>
<td>0.91</td>
</tr>
<tr>
<td>A</td>
<td>1.55%</td>
<td>0.156%</td>
<td>1.23</td>
</tr>
<tr>
<td>Baa</td>
<td>4.39%</td>
<td>0.448%</td>
<td>1.94</td>
</tr>
<tr>
<td>Ba</td>
<td>20.63%</td>
<td>2.248%</td>
<td>3.20</td>
</tr>
<tr>
<td>B</td>
<td>43.915</td>
<td>5.618%</td>
<td>4.70</td>
</tr>
</tbody>
</table>

The Credit Spread Puzzle

• Loss-given-default ($L$) is typically around 50%, so – ignoring risk premia – a *typical credit spread* should be around *half the annual default probability*

• This is *far lower* than we observe in practice:

<table>
<thead>
<tr>
<th></th>
<th>$L \times p$ (b.p.)</th>
<th>Av Credit Spread (b.p.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8</td>
<td>123</td>
</tr>
<tr>
<td>B</td>
<td>281</td>
<td>470</td>
</tr>
</tbody>
</table>

• The *jury is still out* on precisely why, but progress is being made (cyclicality so risk premium, liquidity, …)
Default Rates
One-Year Corporate Issuer Default rate: Investment Grade and Sub-Investment Grade (1920-2004)

Source: Moody’s
**Baa-Aaa Spread and One-Year Investment Grade Default Rates, 1920-2004**

Source: Moody’s
Quarterly Default Rate and Four-Quarter Moving Average, 1991–3Q 08
Source: Altman High Yield Bond Default and Return Report
Cyclicality is potentially important
Default Rates are *Cyclical*

**Panel A: Annual Default Rates**

**Panel B: Monthly Baa–Aaa Yield Spreads**

Shaded areas are NBER-dated recessions. For annual data, any calendar year with at least 5 months being in a recession as defined by NBER is treated as a recession year. *Data source:* Moody's.

Default Rate and the Business Cycle

[Graph showing the relationship between Speculative Grade Default Rate and GDP Growth rate over time, from 1984 to 2004.]
NBER Definition of Recession

A recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales. A recession begins just after the economy reaches a peak of activity and ends as the economy reaches its trough. Between trough and peak, the economy is in an expansion. Expansion is the normal state of the economy; most recessions are brief and they have been rare in recent decades.

Source: http://www.nber.org/cycles/recessions.html
Recovery Rates are also *Cyclical*

Data source: Value-weighted average recovery rates for “All Bonds” and “Sr. Unsecured” are from Moody's. “Altman Data Recovery Rates” are from Altman and Pasternack (2006). Shaded areas are NBER-dated recessions.

Cyclicality Increases the Credit Spread

\[ \frac{1}{1 + R_f} \left( \text{RN-Prob(default)} \cdot \text{Loss in Default} \right) \]

Credit risk price discount

\[ = \frac{1}{1 + R_f} \left( \pi_G m_G L_G + \pi_B m_B L_B \right) \]

\(m_s\): stochastic discount factor

- Cyclicality increases credit spread since, in “bad” state:
  - higher market value of losses (stochastic discount factor, \(m_s\), high)
  - higher loss-given-default (\(L\))
This Course…
Modelling Default: Two Approaches

**Structural Approach**
- **Idea:** default occurs as a result of deficiency of financial resources
- Example: \( V(\text{assets}) < V(\text{debt}) \)
- More generally: firm value hits some boundary
- “fundamental value” approach

**Default Intensity Approach**
- Model probability \( f \) of default directly
- In simple models: no relation to firm value
- Relative value approach
- Rather like tossing a coin (but coin comes down on “default” side very rarely)
Default predictors based on structural models fare much better than ratings...
Dollar Denominated (Altman) Default Rate \([t+1]\) Versus Yield Spread\([t]\)

Source: Altman High Yield Bond Default and Return Report

Regression equation:

Default Rate \(= -3.249 + 1.391 \text{ Spread}\)

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-3.2493</td>
<td>0.9238</td>
<td>-3.52</td>
<td>0.002</td>
</tr>
<tr>
<td>Spread</td>
<td>1.3913</td>
<td>0.1778</td>
<td>7.83</td>
<td>0.000</td>
</tr>
</tbody>
</table>

\(R^2 = 69.4\%\) \(R^2(\text{adj}) = 68.3\%\)
Relation between structural and intensity approaches

Capital structure
Long run financial policy
Risk of firm assets
(\textit{structural})

\begin{itemize}
\item Value of firm assets
\item Risk of corporate debt
\item Value of credit derivatives
\end{itemize}

Relative valuation of credit derivatives in terms of bond price (\textit{intensity})

not often?
• We will discuss liquidity issues in some detail

• The goal of understanding models ***WITHOUT LIQUIDITY EFFECTS*** is to get a good grasp of when the model will fail and ***LIQUIDITY EFFECTS MATTER***…
YTM Spread Between High-Yield Bonds and 10-Yr Treasury Notes, 1 Jun 07–31 Oct 08

Source: Altman High Yield Bond Default and Return Report

What is moving?
Risky yield or Risk-free rate?
Flight to quality?